Perturbation principle and Riccati technique in oscillation criteria for half-linear differential equations

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We consider a pair of half-linear second order differential equations

\[ (r(t)\Phi(x'))' + c(t)\Phi(x) = 0, \tag{1} \]

\[ \left[(r(t) + \tilde{r}(t))\Phi(x')\right]'' + (c(t) + \tilde{c}(t))\Phi(x) = 0, \tag{2} \]

where \( \Phi(x) = |x|^{p-2}x, p > 1 \), and \( r, c, \tilde{r}, \tilde{c} \) are continuous functions such that \( r(t) > 0, r(t) + \tilde{r}(t) > 0 \) for large \( t \).

It is known (see [4]) that basic facts of the oscillation theory for linear Sturm-Liouville differential equations (the special case \( p = 2 \) in (1)) extend almost verbatim to (1). Consequently, similar to the linear case, any solution of (1) has either infinitely many or a finite number of zeros in a neighborhood of infinity, and hence (1) can be classified as oscillatory or nonoscillatory.

On the poster we present the results of the joint paper with O. Došlý [1]. We suppose that (1) is nonoscillatory and we examine the influence of the perturbation terms \( \tilde{r}, \tilde{c} \) on the oscillatory nature of (2).

Our results are based on the Riccati technique, which relates nonoscillation of (2) to the solvability of the generalized Riccati equation

\[ w' + c(t) + \tilde{c}(t) + (p - 1)(r(t) + \tilde{r}(t))^{1-q}|w|^q = 0, \quad q = \frac{p}{p - 1}, \tag{3} \]

in a neighborhood of infinity. The main idea we use is based on comparing the generalized Riccati equation (3) (which is related to (2)) with the Riccati equation associated to a certain linear equation. This connection between linear and half-linear equations enables to use results of linear oscillation theory when investigating (2).

Note that a similar problem was studied in several papers, where the case \( \tilde{r} = 0 \) was investigated, i.e., the perturbation was considered only in the \( c \)-term, see, e.g., [2, 3, 5] and other references of the paper [1]. The motivation for investigating the perturbations in the term involving derivative comes from the paper [6] dealing with linear Sturm-Liouville differential equations.

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References


