Sharp results on growth and decay rates in population model equations

István Győri

University of Pannonia, Veszprém, Hungary

In this paper we investigate the exact growth/decay rate of solutions of age independent and age dependent population model equations.

First part of the lecture is devoted to the study of the age independent logistic equation with infinite delay

$$\dot{N}(t) = N(t) \left\{ r - aN(t) - \int_0^\infty b(s)N(t-s)ds \right\}, \quad t \ge 0,$$

under conditions that force its solutions to approach a positive steady state at large times (see [1]). It is shown that this rate of convergence depends on the initial history in some cases, and is independent of the history in others.

The second part of the lecture is dealing with the so-called a synchronous exponential growth/decay property of some age-structured population models which may be transformed into the non-linear abstract integral equation

$$N(t) = T(t)u + \int_0^t T(t-s)F(x(s))ds, \quad t \ge 0,$$

where T(t) is a strongly continuous semigroup of bounded operators in a Banach space (see [2]).

Examples are given to illustrate the sharpness of the results.

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References

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