Critical oscillation constant for half-linear differential equations with periodic coefficients

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We study oscillation properties of half-linear equations

$$[r(t)\Phi(x')]' + \frac{\gamma c(t)}{t^p}\Phi(x) = 0 \tag{1}$$

and

$$[r(t)\Phi(x')]' + \frac{1}{t^p}\left[\gamma c(t) + \frac{\mu d(t)}{\log^2 t}\right]\Phi(x) = 0, \tag{2}$$

where $\Phi(x) = |x|^{p-1}\operatorname{sgn} x, p > 1$, $\gamma, \mu \in \mathbb{R}$, and r, c, and d are α-periodic positive functions defined on $[0, \infty)$. These equations were studied in the case of constant functions r, c, and d (so-called half-linear Euler and Riemann–Weber equations, respectively) e.g. in [2, 3]. These equations with constant coefficients are conditionally oscillatory, i.e., equation (1) is oscillatory if and only if $\gamma > \frac{r\gamma_p}{c}$, $\gamma_p := \left(\frac{p-1}{p}\right)^p$, and equation (2) is oscillatory for $\gamma > \frac{r\gamma_p}{c}$, non-oscillatory for $\gamma < \frac{r\gamma_p}{c}$, and for $\gamma = \frac{r\gamma_p}{c}$ it is oscillatory if and only if $\mu > \frac{r\mu_p}{d}$, $\mu_p := \frac{1}{2}\left(\frac{p-1}{p}\right)^{p-1}$. The case of periodic coefficients was studied in [5, 6] in the linear case $p = 2$, and the first step to half-linear equations was done in [4], where the oscillation constant of equation (1) was computed.

On the poster, we present the results of [1], where we have computed explicitly the oscillation constants for the equations (1) and (2). The main result is given in the following theorem.

Theorem 1. *Let r, c, d in (1) and (2) be α-periodic positive functions. Equation (1) is non-oscillatory if and only if*

$$\gamma \leq \gamma_{rc} := \frac{\alpha^p \gamma_p}{\left(\int_0^\alpha r^{1-q}(t)\,dt\right)^{p-1}\int_0^\alpha c(t)\,dt}, \quad \gamma_p := \left(\frac{p-1}{p}\right)^p.$$

In the limiting case $\gamma = \gamma_{rc}$, equation (2) is non-oscillatory if

$$\mu < \mu_{rd} := \frac{\alpha^p \mu_p}{\left(\int_0^\alpha r^{1-q}(t)\,dt\right)^{p-1}\int_0^\alpha d(t)\,dt}, \quad \mu_p := \frac{1}{2}\left(\frac{p-1}{p}\right)^{p-1},$$

and it is oscillatory if $\mu > \mu_{rd}$.

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