Picone-type identities in comparison theory of nonlinear equations

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Let \( x(t), G(x, t) \) and \( P(x, t) \) be of the class \( C^1 \) with \( P(x, t) > 0 \). Then

\[
(\alpha + 1) \frac{d}{dt} \left\{ G(x(t), t) \right\} = P |x'|^{\alpha + 1} + \alpha P^{-\frac{\alpha}{\alpha + 1}} \left| \frac{\partial G}{\partial x} \right|^{1 + \frac{\alpha}{\alpha + 1}} + (\alpha + 1) \frac{\partial G}{\partial t} -
\]

\[
P \left[ |x'|^{\alpha + 1} - (\alpha + 1)P^{-1}x \frac{\partial G}{\partial x} + \alpha P^{-\frac{\alpha + 1}{\alpha}} \left| \frac{\partial G}{\partial x} \right|^{1 + \frac{1}{\alpha}} \right] \]

It is shown how this general Picone-type formula can be used for various particular choices of \( P \) and \( G \) to obtain comparison results for various classes of nonlinear differential equations, including half-linear differential equations of the second order and the first-order nonlinear equation governing the motion of a braked cart rolling down an inclined plane.

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