Bubbles, chaos, and the Hydra effect in delayed population models

Eduardo Liz
Vigo, Spain

We study the effects of increasing harvesting in a population model whose dynamics is governed by the delay-differential equation with negative feedback

\[ x'(t) = -\delta x(t) + f(x(t - \tau)), \quad (1) \]

where \( x(t) \) is the number of adult members of the population at time \( t \), \( \delta \) is the natural mortality rate, and \( f \) is the so-called stock-recruitment relationship. It is assumed that there is a constant maturation period \( \tau \), and a constant probability of survival to adulthood.

The most famous population model of this type was suggested by Gurney, Blythe and Nisbet [1], and it is known as the Nicholson’s blowflies equation:

\[ x'(t) = -\delta x(t) + px(t - \tau)e^{-\beta x(t - \tau)}, \quad \delta, p, \beta, \tau > 0. \quad (2) \]

Assuming a strategy of constant effort harvesting prior to reproduction, we consider the following modification of equation (1):

\[ x'(t) = -(\delta + \gamma)x(t) + f((1 - \gamma)x(t - \tau)), \quad (3) \]

where \( \gamma \in (0, 1) \) is the rate of harvesting effort.

We analyze the changes in the dynamics of the solutions of (3) when \( \gamma \) is increased from 0 to 1, taking as a case of study the Nicholson’s blowflies equation (2). The most interesting phenomena that we report are:

- The existence of bubbles in the bifurcation diagram, which are characterized by two consecutive Hopf-bifurcation points,
- a route of period-doubling bifurcations to chaos, and
- the Hydra effect, that means that a population can increase its abundance in response to an increase in its per-capita mortality rate (see [2]).

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References
