Periodic solutions of generalized ordinary differential equations

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Generalized ordinary differential equations (GODE), introduced in 1957 by Jaroslav Kurzweil [1], have been one of the main topics of interest of Stefan Schwabik. Besides many original research papers, Schwabik has written several monographs, and in particular the well known *Generalized Ordinary Differential Equations* published by World Scientific in 1992 [2].

The aim of this joint work with Marcia Federson is to extend to generalized ordinary differential equations

$$\frac{dx}{d\tau} = DF(x,t) \tag{1}$$

some methods for proving the existence of periodic solutions based upon degree theory. Recall that a solution x of the generalized differential equation (1) is a function x such that, for any $s, s' \in [0,T]$ one has

$$x(s) - x(s') = \int_{s'}^{s} DF(x(\tau),t) \tag{2}$$

where the integral in the right-hand member is a generalized Perron integral in the sense of Kurzweil. A *T-periodic solution* of the GODE (1) is a solution of (2) such that $x(0) = x(T)$.

Using a suitable fixed point characterization of the T-periodic solutions of (1), and topological degree arguments, we prove, under suitable regularity conditons upon the function $F : \mathbb{R}^n \times [0,T]$ introduced in Schwabik's monograph **[?]**, the following existence theorem for T-periodic solutions in the space $BV([0,T],\mathbb{R}^n)$.

Theorem 1. *Assume that there exists an open bounded set $\Delta \subset \mathbb{R}^n$ such that the following conditions hold.*

1. *For any $\lambda \in (0,1)$, the GODE*

$$\frac{dx}{d\tau} = \lambda F(x,t) \tag{3}$$

has no T-periodic solution x in BV such that $x \in \Delta$

2. *Equation*

$$\Phi(a) := \int_0^T DF(a,t) = 0 \tag{4}$$

has no solution $a \in \Delta \cap \mathbb{R}^n$ (where $\mathbb{R}^n$ means the set of constant functions in BV).

3. $d_B[\Phi, \Delta \cap \mathbb{R}^n] \neq 0$.

Then the GODE (1) has at least one T-periodic solution $x \in \Delta$.

Applications are given to various special classes of GODE.

References

[1] Kurzweil, Jaroslav, Generalized ordinary differential equations and continuous dependence on a parameter, Czechoslovak Math. J. 7 (82) 1957 418–449.

[2] Schwabik, Stefan, Generalized Ordinary Differential Equations, World Scientific, Singapure, 1992