Stability and estimation of solutions of linear differential systems of neutral type with constant coefficients

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We investigate the exponential-type stability of linear neutral delay differential systems with constant coefficients using Lyapunov-Krasovskii type functionals, more general than those reported in the literature. Delay-dependent conditions sufficient for the stability are formulated in terms of positivity of auxiliary matrices. The approach developed is used to characterize the decay of solutions (by inequalities for the norm of an arbitrary solution and its derivative) in the case of stability, as well as in a general case.

We give estimations of solutions of linear systems neutral differential equations with constant coefficients and a constant delay

$$\dot{x}(t) = D\dot{x}(t-\tau) + Ax(t) + Bx(t-\tau) \tag{1}$$

where $t \geq 0$ is an independent variable, $\tau > 0$ is a constant delay, A, B and D are $n \times n$ constant matrices and $x\colon [-\tau, \infty) \to \mathbb{R}^n$ is a column vector-solution. The derivative " $\cdot$ " is understand as a left-hand derivative.

We use Lyapunov-Krasovskii functionals of a quadratic type depending on running coordinates as well as on their derivatives

$$V_0[x(t), t] = x^T(t)Hx(t) + \int\limits_{t-\tau}^{t} e^{-\beta(t-s)} \left[x^T(s)\, G_1 x(s) + \dot{x}^T(s) G_2 \dot{x}(s)\right] ds$$

and

$$V\left[x(t), t\right] = e^{pt} V_0[x(t), t]$$

where x is a solution of (1), β and p are real parameters, $n \times x$ matrices H, G_1 and G_2 are positively definite, and $t > 0$.

Although many approaches in the literature are used to judge the stability, our approach, except others, not only determines whether the system (1) is exponentially stable but also gives delay dependent estimation of solutions in terms of norms for both $\|x(t)\|$ and $\|\dot{x}(t)\|$ even in the case of instability.

Estimation of the norm of a derivative $\|\dot{x}(t)\|$ is achieved by reducing initial neutral system (1) to a neutral system having the same solution on indicated intervals in which the "neutrality" is contained only on the initial interval.

In the literature, if estimations of solutions are given, then as a rule estimations of derivatives are not investigated.

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