PROBABILITY AND MEASURE THEORY ON TIME SCALES

Berenice C. Damasceno, Luciano Barbanti

State São Paulo University (UNESP), Ilha Solteira, SP, 15385-000, BRAZIL

In 1990, S. Hilger introduced the Calculus on time scales (or on measure chains), [1]. This kind of calculus showed the possibility to manage concepts considering a very wide range of time scales (defined as a closed non-void subset of the real numbers) . Examples of time scales are the real numbers, the integers the sets having cluster points or even such as a Cantor set. The measure theory adapted to time scales, the delta and nabla measures were first defined by Guseinov in 2003, [2]. In a further study, the relationship between Lebesgue delta-integral and Riemann delta-integral on time scales were introduced in detail by Guseinov and Bohner [3]. In 2004, Cabada and Vivero [4] established the relationship between delta-measure and the classical Lebesgue measure, and further between the Lebesgue delta-integral and the classical Lebesgue integral,considering general time scales . The delta-measurability of sets was studied by Rzezuchovsky in 2005, [5], and finally the Lebesgue-Stieltjes measure has been constructed on time scales and the connection between Lebesgue-Stieltjes measure and Lebesgue-Stieltjes delta-measure and also the link between Lebesgue-Stieltjes delta-integral and Lebesgue-Stieltjes integral was done by A.Deniz [6] in 2007.

In this frame we considered more advanced problems in the theory and we propose in this work the consideration of the renewal theorem and the local central limit theorem that works well when considering centered lattices. In this way we established the main theorem solving on time scales, the known in literature as the "occupation-time problem" (see [7]) that determines in a stochastic process the amount of time (in terms of the first trials) that the sums of independent identically distributed tagged random variables spend in the total interval T considered.

References

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