Variational measures in the theory of integration

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We will present here some results concerning the variational measures associated to a real valued function, or, in a more general setting, to a vector valued function. Roughly speaking, given a function Φ defined on an interval $[a, b]$ of the real line it is possible to construct, using suitable families of intervals, a measure μ_Φ which carries information about Φ. If Φ is a real valued function, then the σ-finiteness of the measure μ_Φ implies the a.e. differentiability of Φ, while the absolute continuity of the measure μ_Φ characterizes the functions Φ which are Henstock-Kurzweil primitives. The situation becomes more complicate if we consider functions taking values in an infinite dimensional Banach space. If the Banach space has the Radon-Nikodým property, then it is possible to obtain properties similar to those of the real case. But it is surprising that by means of the variational measures it is possible to characterize the Banach space having the Radon-Nikodým property.

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