Picone-type identities in comparison theory of nonlinear equations

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Let x(t), G(x, t) and P(x, t) be of the class C^1 with P(x, t) > 0. Then

$$(\alpha+1)\frac{d}{dt}\left\{G(x(t),t)\right\} = P|x'|^{\alpha+1} + \alpha P^{-\frac{1}{\alpha}} \left|\frac{\partial G}{\partial x}\right|^{1+\frac{1}{\alpha}} + (\alpha+1)\frac{\partial G}{\partial t} - P\left[|x'|^{\alpha+1} - (\alpha+1)P^{-1}x'\frac{\partial G}{\partial x} + \alpha P^{-\frac{\alpha+1}{\alpha}} \left|\frac{\partial G}{\partial x}\right|^{1+\frac{1}{\alpha}}\right]$$

It is shown how this general Picone-type formula can be used for various particular choices of P and G to obtain comparison results for various classes of nonlinear differential equations, including half-linear differential equations of the second order and the first-order nonlinear equation governing the motion of a braked cart rolling down an inclined plane.

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