The three-dimensional structures formed by monoidal categories, bicategories, double categories, etc.

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Abstract.

Conventional wisdom in category theory suggests that weak n-categories assemble into weak $(n+1)$-categories. Conventional wisdom in architecture suggests that cubes assemble into three-dimensional structures more easily than spheres. It therefore seems only natural to conclude that weak double categories ought to assemble into a weak triple category. To the chagrin of category theorists and architects alike, this is not the case.

Deciding that composition is not such an important aspect of category theory after all, we investigate the three-dimensional structure that double categories do form, and are led inexorably to the concept of a *virtual triple category* [1]. In this talk, I will explain how weak double categories (and, more generally, virtual double categories) assemble into a virtual triple category. Far from being of interest only to double category theorists, I will show that, by restricting our consideration to various virtual triple subcategories – such as those spanned by monoidal categories, bicategories, and multicategories – we reveal the three-dimensionality inherent to these familiar structures, thereby shedding light on phenomena in two-dimensional and enriched category theory [2, 3, 4, 5, 6, 7].

References

[1] Tom Leinster. Generalized enrichment for categories and multicategories, 1999.

[2] Max Gregory Kelly, Anna Labella, Vincent Schmitt, and Ross Street. Categories enriched on two sides. *Journal of Pure and Applied Algebra*, 168(1):53–98, 2002.

[3] J. R. B. Cockett, J. Koslowski, and R. A. G. Seely. Morphisms and modules for poly-bicategories. *Theory and Applications of Categories*, 11(2):15–74, 2003.

[4] J. R. B. Cockett, J. Koslowski, R. A. G. Seely, and R. J. Wood. Modules. *Theory and Applications of Categories*, 11(17):375–396, 2003.

[5] Craig Pastro and Ross Street. Doubles for monoidal categories. *Theory and Applications of Categories*, 21, 2008.

[6] Robert Paré. Yoneda theory for double categories. *Theory and Applications of Categories*, 25(17):436–489, 2011.

[7] Christian Williams. *The Metalanguage of Category Theory*. PhD thesis, University of California, Riverside, 2023.