Cartesian monoidality of the cubical Joyal model structure

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Abstract.

Modelling homotopy theory using cubical sets requires a suitable monoidal product, with respect to which homotopies can be defined. For less structured varieties of cubical sets, such as those having only faces, degeneracies and connections, the cartesian product is often difficult to work with; thus it is more common to use the geometric product when modelling homotopy theory using these cubical sets. This has the advantage of an elegant description, as in contrast to the cartesian product, the geometric product of cubes is again a cube, but has the disadvantage that it is not symmetric, leading to two distinct constructions of mapping spaces between cubical sets. In this talk, based on the paper [2], we will discuss a proof that the cubical Joyal model structure on cubical sets with connections (constructed in [1]), which models $(\infty, 1)$-categories, is monoidal with respect to the cartesian product. This is done by means of a comparison with more structured cubical sets having not only connections, but also symmetries and diagonals, which also allows for the construction of a Quillen-equivalent model structure on the latter category. Moreover, this comparison also allows us to obtain a new proof that the geometric product of cubical sets is symmetric up to natural weak equivalence, even in the absence of connections.

References

[1] B. Doherty, K. Kapulkin, Z. Lindsey, and C. Sattler, *Cubical models of* $(\infty, 1)$*-categories*, Mem. Amer. Math. Soc.$\tilde{2}$97 (2024), no. 1484, v+110 pp.

[2] B. Doherty, Symmetry in the cubical Joyal model structure, preprint arXiv:2409.13842, 2024.