# Linearly Distributive Fox Theorem

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**Abstract**.

Linearly distributive categories (LDC), introduced by Cockett and Seely to model multiplicative linear logic, are categories equipped with two monoidal structures that interact via linear distributivities [2]. A seminal result in monoidal category theory is the Fox theorem, which characterizes cartesian categories as symmetric monoidal categories where each object is equipped with a comonoid structure [3]. The aim of the current work is to extend the Fox theorem to LDCs and characterize the subclass of LDCs, whose tensor structure is cartesian and parr structure is cocartesian, known as cartesian LDCs.

As we will discuss in this talk, we must restrict our attention to LDCs which additionally satisfy the medial logical rule, named medial linearly distributive categories. The medial rule has appeared frequently in various deep inference systems [5]. It has equally been crucial in certain developments of categorical semantics for classical logic [4, 6]. Within monoidal category theory, the medial rule is better known as an instance of the interchange law of duoidal categories [1]. Indeed, medial LDCs can be thought of as the appropriate structure at the intersection of LDCs and duoidal categories.

Mirroring the cocommutative comonoids of the Fox theorem, a medial LDC induces a cartesian LDC by constructing the category of biccommutative medial bimonoids. The concept of medial linear functors, and medial linear transformations are equally introduced and an adjunction between the 2-categories of medial and cartesian LDCs is proved, named the *linear distributive Fox theorem*.

## References

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