

Upgrading equivalences in a weak ω -category to coherent ones

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Abstract.

It is well known that any equivalence in a 2-category can be upgraded to an adjoint equivalence. I will present a weak ω -categorical version of this result.

More precisely, consider an ω -functor $F: X \rightarrow Y$ between weak ω -categories in the sense of Batanin [1] and Leinster [3]. We say F is an ω -*equifibration* if, for any n -cell x in X and for any (coinductive) equivalence $(n+1)$ -cell $u: Fx \rightarrow y$ in Y , there exists an equivalence $(n+1)$ -cell $\bar{u}: x \rightarrow \bar{y}$ in X such that $F\bar{u} = u$; this is an ω -dimensional analogue of isofibrations between ordinary categories. This definition cannot be directly translated into a right lifting property because it involves the *property* (as opposed to a *structure*) of u and $\bar{u}$ being equivalences. In fact the model categorical intuition tells us that, in order to characterise the ω -equifibrations by a right lifting property, we would need a notion of *coherent* equivalence. Our main result provides two such notions, namely that of *coinductive half-adjoint equivalence* and that of *coinductive equivalence with separate left and right inverses*.

We also prove that the ω -equifibrations between *strict* ω -categories are precisely the fibrations in the folk model structure constructed in [2], leading to an explicit description of generating folk trivial cofibrations (which involves the strict ω -category $\omega\mathcal{E}$ of [4]).

References

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