

Eckmann-Hilton Arguments in Weak ω -categories

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Abstract. We prove generalisations of the Eckmann-Hilton argument in the context of globular weak ω -categories [6]. Given a strict monoidal category, the classical Eckmann-Hilton argument [2] shows that, for any endomorphisms a, b of the monoidal unit:

$$a \circ b = a \otimes b = b \circ a$$

This argument immediately generalises to strict higher categories to show the commutativity of all compositions of cells with sufficiently degenerate boundary. We further generalise this to weak ω -categories, producing a family of equivalences witnessing these commutativity results. In this setting, the proofs become significantly more complex. In particular, $a \circ b$ and $a \otimes b$ are no longer parallel due to weak unitality. This forces us to introduce a *padding* construction, similar to those of Finster et al. [3] and Fujii et al. [5], to compare non-parallel cells. The work has been implemented in the proof assistant CATT [4] for ω -categories, and can be exported to HOTT [1].

References

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