A categorical perspetive on the complexity of satisfying constraints

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Abstract.

The so-called algebraic approach to the *constraint satisfaction problem (CSP)* has been a prevalent method of the study of complexity of these problems since early 2000's. The core of this approach is the notion of *polymorphisms* which determine the complexity of the problem (up to log-space reductions). This theory has started with the work of Jeavons, Cohen, and Gyssens [3], and has been further developed over the past 3 decades including a generalisation of the scope to the *promise constraint satisfaction problem (PCSP)* [2]. Nevertheless, recent work also suggests that insights from other fields are immensely useful in the study of PCSPs including algebraic topology (see, e.g., [4]).

The aim of the talk is to provide a gentle introduction for category-theorists into the study of complexity of CSPs and PCSPs [1]. We show that many standard CSP notions have clear and well-known categorical counterparts. For example, the algebraic structure of polymorphisms can be described as a set-functor defined as a right Kan extension. We provide purely categorical proofs of core results of the algebraic approach including a proof that the complexity only depends on the polymorphisms. Our new proofs are substantially shorter and, from the categorical perspective, cleaner than previous proofs of the same results. Moreover, as expected, are applicable more widely. We believe that, in particular in the case of PCSPs, category theory brings insights that can help solve some of the current challenges of the field.

References

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