

A higher categorical approach to the André-Quillen cohomology of an $(\infty, 1)$ -Category

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Abstract.

Simplicial categories, that is categories enriched in simplicial sets, are a model of $(\infty, 1)$ -categories. Their André-Quillen cohomology, originally introduced by Dwyer and Kan [DK2], was later re-interpreted and extended by Harpaz, Nuiten and Prasma [HNP1]. The André-Quillen cohomology of a simplicial category can be used to describe its k -invariants which in turn contain various higher homotopy information and in particular yield an obstruction theory for realizing homotopy-commutative diagrams [DK2].

Our aim is to give an algebraic, elementary and explicit approach to the André-Quillen cohomology of simplicial categories using the tools of higher category theory. For this purpose, we first observe that in order to study the n^{th} André-Quillen cohomology group of a simplicial category, it suffices to look at simplicial categories that are n -truncated, that is they are enriched in n -types. This has the advantage that we can use one of the algebraic models of n -types from higher category theory to produce an algebraic replacement for the n^{th} Postnikov truncation of a simplicial category.

We choose to use the category $\mathbf{GCat}_{\text{wg}}^n$ of groupoidal weakly globular n -fold categories arising within Paoli's model of weak n -categories [Pa3]. This category is a model of n -types with a cartesian monoidal structure. Further, every n -type can be modelled by a weakly globular n -fold groupoid, that is an object of the full subcategory $\mathbf{Gpd}_{\text{wg}}^n$ of $\mathbf{GCat}_{\text{wg}}^n$ [BP2], which is more convenient algebraically. Our model for the n^{th} Postnikov truncation of a simplicial category is a category enriched in $\mathbf{Gpd}_{\text{wg}}^n$ with respect to the cartesian monoidal structure. We call the latter an n -track category.

Using the n -fold nature of $\mathbf{Gpd}_{\text{wg}}^n$ we iteratively build a comonad on n -track categories. Using this comonad we then obtain an explicit cosimplicial abelian group model for the André-Quillen cohomology of an $(\infty, 1)$ -category. This is joint work with David Blanc [BP4].

References

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