

The higher algebra of monoidal bicategories

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Abstract.

The aim of this talk is to relate precisely low-dimensional category theory and higher-dimensional category theory and to provide some applications of the latter to the former. We shall be particularly interested in the theory of monoidal bicategories, as developed by Day and Street in [1], and higher algebra, as developed by Lurie in [3]. A key difference between the two subjects is that one focuses on fully algebraic notions, while the other is essentially homotopical in nature, being part of the theory of $(\infty, 1)$ -categories. Yet, as we will see, the theory of E_n -operads, which originates from the classification of n -fold loop spaces [4], provides a convenient way to handle concepts in both areas.

Our first main result shows that braided, sylleptic and symmetric monoidal bicategories, in the sense of [1], are exactly E_n -algebras, for $n = 2, 3, 4$, respectively, in the sense of [3]. The proof of this result involves introducing an $(\infty, 1)$ -category of bicategories and subtle considerations on the interaction between strictification results and homotopy-coherent associative structures known as A_n -algebras.

As an application, we give a conceptual proof of the bicategorical version of the fact that monoids in a (symmetric) braided monoidal category form a (braided, and in fact symmetric) monoidal category [2]. For a monoidal bicategory $\mathcal{E}$, there is a bicategory $\mathbf{PsMon}(\mathcal{E})$ of pseudomonoids in $\mathcal{E}$ and

- if $\mathcal{E}$ is braided, then $\mathbf{PsMon}(\mathcal{E})$ is monoidal;
- if $\mathcal{E}$ is sylleptic, then $\mathbf{PsMon}(\mathcal{E})$ is braided;
- if $\mathcal{E}$ is symmetric, then $\mathbf{PsMon}(\mathcal{E})$ is sylleptic, and in fact symmetric.

This result was proved by Nick Gurski (and, independently, by Nicola Gambino) by direct calculations. Here instead, thanks to our first main result, we prove this result using the homotopy-coherent tools of higher algebra and hence largely avoid the daunting pasting diagrams that are typical of the theory of monoidal bicategories. In particular, the result follows from the Dunn Additivity Theorem for E_n -algebras proved by Lurie.

References

- [1] B. Day and R. Street, *Monoidal Bicategories and Hopf Algebroids*, 1997.
- [2] A. Joyal and R. Street, *Braided tensor categories*, 1982.
- [3] J. Lurie, *Higher Algebra*, 2017.
- [4] P. May, *The Geometry of Iterated Loop Spaces*, 1972.