8TH INTERNATIONAL CONFERENCE ON DIFFERENCE
EQUATIONS AND APPLICATIONS
JULY 28 – AUGUST 1, 2003, BRNO, CZECH REPUBLIC

ICDEA 2003, Masaryk University, Faculty of Science, Janáčkovo nám. 2a, 66295 Brno, Czech Republic, e-mail: icdea2003@math.muni.cz, Phone: +420-5-41321251, Fax: +420-5-41210337

ICDEA 2003

organized by

the Faculty of Science, Masaryk University Brno
the Faculty of Civil Engineering, University of Technology Brno
the Mathematical Institute of the Academy of Sciences CR

in cooperation with

the Union of Czech Mathematicians and Physicists
the Union of Slovak Mathematicians and Physicists

under the auspices of

Petr Duchoň, the Mayor of the City of Brno
Helena Illnerová, president of the Academy of Sciences CR
the International Society of Difference Equations

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is highly appreciated.

Dear Colleague and Guest,

Welcome to Brno, the center of the Moravian region of the Czech Republic and the venue of the 8th International Conference on Difference Equations and Applications (ICDEA 2003).

This conference is the eighth meeting in a series of conferences which started in San Antonio, Texas/USA in 1994. The consecutive meetings took place in Veszprém/Hungary (1995), Taipei/Taiwan (1997), Poznań/Poland (1998), Temuco/Chile (2000), Augsburg/Germany (2001), Changsha/China (2002).

This booklet contains the information which should help you to get acquainted with both the scientific and the social program of the conference. The organizing committee will do their best to make your stay in Brno both fruitful and pleasant. The scientific program was prepared by the scientific committee, but the conference could not be successful without the active cooperation of all its participants. We are especially grateful to invited speakers for accepting our invitation. We thank all participants for their contributions to the scientific program, and we beg those whom we asked to chair one of the sessions to consider it a significant service to the organizers.

Continuing in style and spirit of the previous conferences, also the eighth meeting covers all themes of the field of ordinary and partial difference equations, classical and contemporary as well as theoretical and applied. Apart from 11 plenary lectures (50 minutes) the present conference will have 7 invited lectures (30 minutes) and about 60 contributed talks (20 minutes).

We do hope you will meet old friends in Brno, as well as make new ones, will enjoy your stay in Brno, and leave with many stimuli for your future research.

Organizing Committee ICDEA 2003

Contents

PROGRAM

General Information

Conference site

The conference is held at the building of the Faculty of Civil Engineering, University of Technology Brno, Veveří street 95. There are five lecture rooms (D1, D2 in the building D; C1, C2, C3 in the building C) and aula (in the building A).
A member of our conference office will pick up the participants at the hotel Continental and the hostel Družba on Monday (July 28), 9:00 am, in order to show how to get to the conference site.

Conference office

The conference office is located

- at the hotel Continental (Su, July 27, 17:00–21:00),
- at the hostel Družba (Su, July 27, 18:00–20:00),
- in the building D (Mo, Tu, Thu: 8:30–18:00; We, Fri: 8:30–13:00)

The conference staff can be recognized by blue badges. If you have any questions, please feel free to ask any member of the conference staff. Among others, the tickets for lunches, the tickets for the Farewell Party, and the conference T-shirts can be obtained in the conference office.

Meals

The University Cafeteria where lunches are served (building A) is working on Mon, Tue, Thu, Fri from 12:30 to 14:00, and on Wed from 12:00 to 13:00. The

tickets for the lunches can be bought either as a package for the whole week (the total price 20 EURO or 650 CZK), or separately, the day before each lunch (till 11 am) for 130 CZK, at the conference office.

For the participants who stay at the hostel Družba, breakfasts are served at the cafeteria of the neighboring hostel "Listovy koleje", daily from 7:30 to 8:45.

Of the great variety of restaurants close to the conference site, we recommend the restaurant *Lucerna* (close to the hotel Continental, Slovákova street), the restaurant *Plzeňská pivnice* (close to the hotel Continental, Žerotínovo náměstí), and the restaurant *Plzeňský dvůr* (behind the hostel Družba). We recommend also the restaurant-small brewery *Pegas* (Jakubská street) where their own special wheat beer is served.

Refreshments

Coffee, tea, mineral water, cakes and cookies will be served (free of charge) in the main lobby of the building D during the morning and afternoon coffee breaks.

E-mail

Computers with network access are available in the conference office. Information on how to use these computers for your e-mail can be obtained in the conference office.

Tickets for public transport

Tickets for public transport are available in the conference office. The price is 7 CZK for "10 minutes" ticket, 12 CZK for "40 minutes" ticket. d **Social Program**

During the conference week the following social events will take place:

- Monday (July 28): **Welcome party** with folk music and wine tasting – wines from the South Moravia – will take place in the University Cafeteria (building A) at 19:30 (free of charge for registered participants and accompanying persons). A little joyful lottery is also prepared.

- Wednesday (July 30): **The trips to the Moravian Karst and Lednice Castle**. The departure is at 13:30 from the hotel Continental and at 13:40

from the hotel Družba. The arrival is supposed at 19:00. The trip is free of charge for registered participants and accompanying persons.

- **Moravian Karst:** Though Macocha is not the deepest abyss, it is undoubtedly the most attractive for tourist. It is open to public throughout the whole year. The bottom of the abyss is impressively spacious (174×76 m) and even enlarges in the underground where we could find further caverns. The dramatic panorama is highlighted by two small lakes at the bottom of the abyss in the depth of 138.7 meters, which are linked by short rapids of the underground the Punkva river. It is possible to overview the bottom from two viewpoints at different levels above the bottom; it is possible to use suspended cableway to get to these. There is a walking path at the bottom of the abyss and it is combined with a boat-cruise on the Punkva river.
  Please remember that there is quite cold in the caves, about 5° C. The participants are kindly asked to be punctual. The buses cannot wait; the tour in the caves starts at 15:00.
- **Lednice:** The complex is located south of Brno between the towns of Lednice and Valtice close to the Austria border. It covers approximately an area of 2 km squared. The whole complex is an excellent example of a landscape changed into a piece of art. The complex was owned by the house of Liechtenstein from the second half of the 14th century till 1945. They created, especially out of Lednice chateau, a noble representative manor house. It was in the 19th century when the chateau was rebuilt in the English Neo-gothic style, which we can admire up to these days.

  Around the castle French gardens and an English park were established. The park has been gradually changed into an open countryside with a complex of artificial lakes and ponds since the 15th century. Small romantic structures were built in the surrounding woods (Minaret, Castle of Jan, Rendez-vous, Lake's Chateau, Border's Chateau, Colonnade, Church of Apollo, Three Graces, etc.). The whole region is an important vine county. In December 1996 the Lednice-Valtice complex was included in the UNESCO's "World's Cultural Heritage List".

- Thursday (July 31): **Farewell party** will take place at the garden restaurant *Na Střelnici* at 19:30 (the price for this party, including dinner, is 20 EURO). The bus will go from the hotel Continental at 19:00. The place

can also be reached by the tram No. 1 , which goes from the Česká street (close to the hotel Continental) or by the trolley-buses 145, 146, which go from the Kotlářská street (close to the conference site). You should get off at the stop "Pisárky", then cross the river. The restaurant is on your left, close to the museum "Anthropos". The party starts in the "Imperial-Royal style" by a cannonade. The tasting of traditional drinks (slivovice, becherovka), wines from the South Moravia, and pigling barbecue are also included.

For the participants who are interested in playing tennis, we can provide all necessary equipment. The short excursion to the Observatory of the Institute of Geodesy located in the building C is planned for Thursday at 14:00. Please ask in the conference office for details.

Program for accompanying persons

Sightseeing tour is planned on Tuesday, July 29. The departure will be at 9:30 am from the hotel Continental. The arrival is supposed at 12:00.

ISDE Awards

The international Society of Difference Equations (ISDE) was officially born during ICDEA 2001 in Augsburg, Germany. At this year conference it is the first time when the ISDE awards the following three prizes:

- The prize for the best one-hour talk
- The prize for the best 20-minute or 30-minute talk
- The prize ($ 500) for the best paper on difference equations in the year 2002.

The winners of the two first prizes will be elected by the participants on Friday. The third will be given by the international publisher Taylor & Francis.

Conference Proceedings

The Conference Proceedings will be published in 2004. We suppose to publish it by the international publisher Taylor & Francis. A copy of the proceedings is included in the registration fee for the regular participants.

All papers submitted to these proceedings are subject to a refereeing process.

The contributed papers are restricted in length 8 pages.

The technical details concerning the preparation of manuscripts will be sent to the contributors by e-mail after the conference. The deadline for submission is October 31, 2003. E-mail for sending contributions: `icdea2003@math.muni.cz`.

Scientific Program

MONDAY – JULY 28

9:30–10:00 **Opening Ceremony (Aula)**

10:00–10:30 *Coffee break*

INVITED LECTURES

Room D1 Chairman: Ladas Gerry

10:30–11:20 **Elaydi Saber** (USA)
Nonautonomous difference equations: A fresh look

11:30–12:20 **Smítal Jaroslav** (Czech Republic)
Triangular maps with zero topological entropy – recent results and open problems

Chairman: Diblík Josef

12:25–12:55 **Bohner Martin** (USA)
An oscillation criterion for delay dynamic equations

SHORT COMMUNICATIONS

Room C1 Chairman: Guseinov Sharif

14:30–14:50 **Domoshnitsky Alexander** (Israel)
Distribution of zeros of solutions to functional equations in the space of essentially bounded functions

14:55–15:15 **Rodriguez Jesus** (USA)
Nonlinear boundary value problems on sequence spaces

15:20–15:40 **Yoshitomi Kazushi** (Japan)
Coexistence problems for the Hill equations with 3-step potentials

15:45–16:15 *Coffee break*

Chairman: Bohner Martin

16:15–16:35 **Răsvan Vladimir** (Romania)
Discrete-time Hamiltonian systems: λ-zones and parametric resonance

16:40–17:00 **Hilscher Roman** (Czech Republic)
What is the true discrete strengthened Legendre condition?

17:05–17:25 **Elyseeva Julia** (Russia)
Shearing transformations for symplectic systems

17:30–17:50 **Dannan Fozi** (Qatar)
TBA

Room C2 Chairman: Krause Ulrich

14:30–14:50 **Camouzis Elias** (Greece)
On the period five trichotomy behavior of positive solutions of the difference equation $x_{n+1} = \frac{p+x_{n-k}}{x_n}$

14:55–15:15 **Grove Edward** (USA)
On the trichotomy character of a difference equation

15:20–15:40 **Petropoulou Eugenia** (Greece)
Solutions in ℓ_2 of linear systems of difference equations

15:45–16:15 *Coffee break*

Chairman: Khusainov Denys

16:15–16:35 **Boichuk Alexander** (Slovak Republic/Ukraine)
Dichotomy and trichotomy on the whole line $\mathbb{Z}$ for perturbed difference systems

16:40–17:00 **Baštinec Jaromír** (Czech Republic)
Initial data generating bounded solutions of linear discrete equations

17:05–17:25 **Kobza Aleš** (Czech Republic)
Oscillation and nonoscillation for two-terms difference equations of the third order

17:30–17:50 **Li Wan-Tong** (People's Republic of China)
Existence of positive periodic solutions of a class of delay difference equations and applications in population dynamics

Room C3 Chairman: Andres Jan

14:30–14:50 **Migda Małgorzata** (Poland)
On a class of fourth order nonlinear difference equations

14:55–15:15 **Schmeidel Ewa** (Poland)
Asymptotic properties of fourth order nonlinear difference equations with quasidifferences

15:20–15:40 **Andruch-Sobiło Anna** (Poland)
Bounded solutions of third order nonlinear difference equations

15:45–16:15 *Coffee break*

Chairman: Pituk Mihály

16:15–16:35 **Schinas Christos** (Greece)
On the system of two diference equations $x_{n+1} = \dfrac{p + y_{n-2}}{y_n},\ y_{n+1} = \dfrac{q + x_{n-2}}{x_n}$

16:40–17:00 **Stefanidou Gesthimani** (Greece)
Asymptotic behavior of the solutions of a fuzzy difference equation

17:05–17:25 **Papaschinopoulos Garyfalos** (Greece)
Trichotomy of a system of two difference equations

17:30–17:50 **Cheung Wing-Sum** (Hong Kong)
Some discrete nonlinear inequalities and applications to boundary value problems

TUESDAY – JULY 29

INVITED LECTURES

Room D1 Chairman: Nijhoff Frank

9:00–9:50 **Sacker Robert J.** (USA)
Global stability of periodic orbits of nonautonomous difference equations and population biology

10:00–10:30 *Coffee break*

Chairman: Sell George R.

10:30–11:20 **Siafarikas Panayiotis** (Greece)
A functional-analytic method for the study of difference equations

Room D1 Chairman: Kratz Werner

11:30–12:00 **Erbe Lynn** (USA)
Linear and nonlinear oscillation theory of dynamic equations on time scales

12:05–12:35 **Winternitz Pavel** (Canada)
Symmetry preserving discretizations of differential equations and their applications as numerical methods

Room D2 Chairman: Wu Shujin

11:30–12:00 **Khusainov Denys** (Ukraine)
Solution stability of systems of difference equations with nonlinearity of special type

12:05–12:35 **Pituk Mihály** (Hungary)
Higher order difference equations generating a monotone discrete dynamical system

SHORT COMMUNICATIONS

Room C1 Chairman: Erbe Lynn

14:30–14:50 **Akın-Bohner Elvan** (USA)
Some dynamic inequalities on time scales

14:55–15:15 **Lawrence Bonita** (USA)
A quasi-linearization technique for a dynamic initial value problem on time scales and thoughts beyond

15:20–15:40 **Pötzsche Christian** (Germany)
A limit set trichotomy for systems on time scales

15:45–16:15 *Coffee break*

Chairman: Čermák Jan

16:15–16:35 **Pospíšil Zdeněk** (Czech Republic)
The Abel dynamic equation

16:40–17:00 **Řehák Pavel** (Czech Republic)
A Hardy inequality and half-linear dynamic equations on time scales

17:05–17:25 **Simon Moritz** (Germany)
The Banach space of positively regressive functions on a time scale

17:30–17:50 **El Manjli Abdelouahed** (Morocco)
TBA

Room C2 Chairman: Schinas Christos

14:30–14:50 **Andres Jan** (Czech Republic)
Sharkovskii's theorem and differential equations

14:55–15:15 **Fachada Jose-Luis, Ferreira Alves João** (Portugal)
On a condition for transitivity of interval maps

15:20–15:40 **Fišer Jiří** (Czech Republic)
Iterated multivalued systems

15:45–16:15 *Coffee break*

Chairman: Camouzis Elias

16:15–16:35 **Fernandes Sara** (Portugal)
Second smaller zero of kneading determinant for iterated maps

16:40–17:00 **Oliveira Henrique** (Portugal)
Iterates of the tangent map - the bifurcation scheme

17:05–17:25 **Vinagre Sandra** (Portugal)
Symbolic dynamics in the time-delayed Chua's circuit

17:30–17:50 **Akinremi Samuel Babatunde** (Nigeria)
Quasi-static growth of brittle fracture: A variational methods based on local minimization

Room C3 Chairman: Balla Katalin

14:30–14:50 **Al-Hassan Qassem** (United Arab Emirates)
On inverses of tridiagonal matrices

14:55–15:15 **Madani Moussai** (Algeria)
Pointwise multiplication in generalized Besov and Lizorkin-Triebel spaces

15:20–15:40 **Pirov Rahmon** (Tajikistan)
On the compatibility conditions and on the manifolds of the solutions of the quasi-linear equation systems of the four differential equations with the three unknown functions in the space

15:45–16:15 *Coffee break*

Chairman: Takano Katsuo

16:15–16:35 **Janglajew Klara** (Poland)
Moments of solutions to a linear difference equation

16:40–17:00 **Ruffing Andreas** (Germany)
Analytic properties of a special q-exponential function

17:05–17:25 **Vafeas Panayiotis** (Greece)
The 3D Happel model for complete isotropic Stokes flow

17:30–17:50 **Kamel Haouam** (Algeria)
Global existence of solution of reaction diffusion systems via Lyapunov functional

WEDNESDAY – JULY 30

INVITED LECTURES

Room D1 Chairman: Doedel Eusebius

9:00–9:50 **Nishimura Kazuo** (Japan)
Discrete time dynamics from optimizing model in economics

10:00–10:30 *Coffee break*

Room D1 Chairman: Siafarikas Panayiotis

10:30–11:20 **Ladas Gerry** (USA)
Open problems and conjectures

11:30–12:00 **Kratz Werner** (Germany)
Nonnegativity of quadratic functionals

Room D2 Chairman: Winternitz Pavel

11:30–12:00 **Pera Maria Patrizia** (Italy)
Local bifurcation of periodic solutions of forced equations on manifolds

THURSDAY – JULY 31

INVITED LECTURES

Room D1 Chairman: Hilger Stefan

9:00–9:50 **Nijhoff Frank** (United Kingdom)
Integrable partial difference equations; a modern perspective

10:00–10:30 *Coffee break*

Room D1 Chairman: Nishimura Kazuo

10:30–11:20 **Doedel Eusebius** (Canada)
Collocate and bifurcate (with differences)

11:30–13:00 **ISDE Meeting (incl. ISDE Awards)**

SHORT COMMUNICATIONS

Room C1 Chairman: Hilscher Roman

14:30–14:50 **Došlá Zuzana** (Czech Republic)
Recessive and dominant solutions for half-linear difference equations

14:55–15:15 **Jaroš Jaroslav** (Slovak Republic)
Discrete Picone's identity and its applications in the comparison theory of nonlinear difference equations

15:20–15:40 **Diblík Josef** (Czech Republic)
Retract principle and positive solutions of discrete equations

15:45–16:15 *Coffee break*

Chairman: Răsvan Vladimir

16:15–16:35 **Čermák Jan** (Czech Republic)
On the related asymptotics of delay differential and difference equations

16:40–17:00 **Wu Shujin** (China)
Stability of impulsive functional differential equations with jumps at stopping times

17:05–17:25 **Chiricalov Vladimir** (Ukraine)
Periodic solutions of nonlinear difference matrix equations with bilinear main part

17:30–17:50 **Pehlivan Serpil** (Turkey)
TBA

Room C2 Chairman: Domoshnitsky Alexander

14:30–14:50 **Guseinov Sharif** (Latvia)
The analytical and numerical method for some inverse heat transfer problems in layered media

14:55–15:15 **Hommel Angela** (Germany)
Discrete Cauchy-Riemann operators in the plane as foundation of an approximation of Navier-Stokes and Vekua Equations

15:20–15:40 **Peřinová Vlasta** (Czech Republic)
Difference equations for photon number distribution in the stationary regime of a random laser

15:45–16:15 *Coffee break*

Chairman: Jaroš Jaroslav

16:15–16:35 **Takano Katsuo** (Japan)
On the Gauss hypergeometric series with roots outside the unit disk

16:40–17:00 **Krause Ulrich** (Germany)
Opinion dynamics in higher dimensions

17:05–17:25 **Gaiko Valery A.** (Belarus)
On a new approach to the limit cycle problem

17:30–17:50 **Mahmudov Elimhan** (Turkey)
TBA

Room C3 Chairman: Schmeidel Ewa

14:30–14:50 **Dobrynskiy Vladimir** (Ukraine)
Unexpected issues from classic mathematical model of population dynamics

14:55–15:15 **Guzowska Małgorzata** (Poland)
Bubble bifurcations in the economic models

15:20–15:40 **Osipov Andrey** (Finland)
On one asymptotic formula for the finite difference equations

15:45–16:15 *Coffee break*

Chairman: Boichuk Alexander

16:15–16:35 **Balla Katalin** (Hungary)
On some discrete problems related to differential-algebraic equations

16:40–17:00 **Hamaya Yoshihiro** (Japan)
Bifurcation of almost periodic solutions in difference equations

17:05–17:25 **Cima Anna** (Spain)
On periodic rational difference equations of order k

FRIDAY – AUGUST 1

INVITED LECTURES

Room D1 Chairman: Smítal Jaroslav

9:00–9:50 **Hilger Stefan** (Germany)
The harmonic oscillator — an extension via measure chains

10:00–10:30 *Coffee break*

Chairman: Sacker Robert J.

10:30–11:20 **Sell George R.** (USA)
Dynamical issues arising from time discretizations

11:30–12:20 **Aulbach Bernd** (Germany)
Nonautonomous dynamical systems: Basic concepts and new results

12:30–xx:xx **ISDE Awards for the Best Lecture, Closing Ceremony**

MONDAY – JULY 28

INVITED LECTURES

	Aula	**Room D1**
9:30–10:00	*Opening Ceremony*	
10:00–10:30	*Coffee break*	
Chairman		*Ladas*
10:30–11:20		Elaydi
11:30–12:20		Smítal
Chairman		*Diblík*
12:25–12:55		Bohner

SHORT COMMUNICATIONS

	Room C1	**Room C2**	**Room C3**
Chairman	*Guseinov*	*Krause*	*Andres*
14:30–14:50	Domoshnitsky	Camouzis	Migda
14:55–15:15	Rodriguez	Grove	Schmeidel
15:20–15:40	Yoshitomi	Petropoulou	Andruch-Sobiło
15:45–16:15	*Coffee break*		
Chairman	*Bohner*	*Khusainov*	*Pituk*
16:15–16:35	Răsvan	Boichuk	Schinas
16:40–17:00	Hilscher	Baštinec	Stefanidou
17:05–17:25	Elyseeva	Kobza	Papaschinopoulos
17:30–17:50	Dannan	Li	Cheung

TUESDAY – JULY 29

INVITED LECTURES

	Room D1	**Room D2**
Chairman	*Nijhoff*	
9:00–9:50	Sacker	
10:00–10:30	*Coffee break*	
Chairman	*Sell*	
10:30–11:20	Siafarikas	
Chairman	*Kratz*	*Wu*
11:30–12:00	Erbe	Khusainov
12:05–12:35	Winternitz	Pituk

SHORT COMMUNICATIONS

	Room C1	**Room C2**	**Room C3**
Chairman	*Erbe*	*Schinas*	*Balla*
14:30–14:50	Akın-Bohner	Andres	Al-Hassan
14:55–15:15	Lawrence	Fachada	Madani
15:20–15:40	Pötzsche	Fišer	Pirov
15:45–16:15	*Coffee break*		
Chairman	*Čermák*	*Camouzis*	*Takano*
16:15–16:35	Pospíšil	Fernandes	Janglajew
16:40–17:00	Řehák	Oliveira	Ruffing
17:05–17:25	Simon	Vinagre	Vafeas
17:30–17:50	El Manjli	Akinremi	Kamel

WEDNESDAY – JULY 30

INVITED LECTURES

	Room D1	**Room D2**
Chairman	*Doedel*	
9:00–9:50	Nishimura	
10:00–10:30	*Coffee break*	
Chairman	*Siafarikas*	*Winternitz*
10:30–11:20	Ladas	
11:30–12:00	Kratz	Pera

THURSDAY – JULY 31

INVITED LECTURES

	Room D1
Chairman	*Hilger*
9:00–9:50	Nijhoff
10:00–10:30	*Coffee break*
Chairman	*Nishimura*
10:30–11:20	Doedel
11:30–13:00	**ISDE meeting**

SHORT COMMUNICATIONS

	Room C1	**Room C2**	**Room C3**
Chairman	*Hilscher*	*Domoshnitsky*	*Schmeidel*
14:30–14:50	Došlá	Guseinov	Dobrynskiy
14:55–15:15	Jaroš	Hommel	Guzowska
15:20–15:40	Diblík	Peřinová	Osipov
15:45–16:15	*Coffee break*		
Chairman	*Răsvan*	*Jaroš*	*Boichuk*
16:15–16:35	Čermák	Takano	Balla
16:40–17:00	Wu	Krause	Hamaya
17:05–17:25	Chiricalov	Gaiko	Cima
17:30–17:50	Pehlivan	Mahmudov	

FRIDAY – AUGUST 1

INVITED LECTURES

	Room D1
Chairman	*Smítal*
9:00–9:50	Hilger
10:00–10:30	*Coffee break*
Chairman	*Sacker*
10:30–11:20	Sell
11:30–12:20	Aulbach
12:30–xx:xx	**ISDE Awards, Closing Ceremony**

ABSTRACTS

Invited Lectures

NONAUTONOMOUS DYNAMICAL SYSTEMS: BASIC CONCEPTS AND NEW RESULTS

Bernd Aulbach, *Augsburg, Germany*

The classical theory of dynamical systems is concerned with time variant phenomena which are governed by rules which do not change in time. While this mathematical theory is well established and successful, it has its limitations when phenomena are studied whose underlying laws of motion depend explicitly on time (e.g. by excitations, controls or noise). An extension of the classical theory to the so-called "nonautonomous" dynamical systems is therefore of vital importance both from a theoretical and practical point of view. In this talk we describe the basic concepts of discrete-time nonautonomous dynamical systems and present some new results on nonautonomous attractors and their numerical approximation.

AN OSCILLATION CRITERION FOR DELAY DYNAMIC EQUATIONS

Martin Bohner, *Rolla, USA*

We present an oscillation criterion for first order delay dynamic equations. This criterion contains well-known oscillation criteria for delay differential equations and delay difference equations. We illustrate our result with several examples.

COLLOCATE AND BIFURCATE (WITH DIFFERENCES)

Eusebius Doedel, *Montreal, Canada*

In this talk I will give an overview of the use of finite element collocation methods for solving boundary value problems (BVPs), in the context of numerical continuation and bifurcation analysis. First, I will give a brief review of the method applied to BVPs in ODEs, with application to the computation of periodic solutions of the Circular Restricted 3-Body Problem of Celestial Mechanics. Secondly, I will discuss some convergence results for delay differential equations, including neutral delay differential equations (both in a boundary value problem setting). Finally, I will describe an extension to elliptic PDEs, including new finite element collocation methods for triangular elements.
Various parts of this work are in cooperation with Randy Paffenroth and Herb Keller (Pasadena CA), Don Dichmann (Torrance, CA), Jorge Galán and coworkers (Sevilla), André Vanderbauwhede (Gent), Koen Engelborghs (Leuven), Hamid Sharifi (Ste-Foy).

NONAUTONOMOUS DIFFERENCE EQUATIONS: A FRESH LOOK

Saber Elaydi, *Texas, USA*

Recent questions in theoretical ecology on the global stability of periodically forced discrete population models led the author and Robert Sacker to develop a theory of periodic difference equations. We extend Elaydi and Yakubu theorem to continuous maps acting on nonconnected metric spaces and then to periodic difference equations. In addition, the theorems of Sharkovsky and Li and Yorke will be extended. In this talk we will introduce the notion of skew-product dynamical systems in its most simplified form and indicate how it would give insights into topics of current interest such as dichotomy, stability, and chaos. Recent work by the author jointly with Aulbach and Zeigler on a fourth-order discrete Schrodinger's equation will be presented.

LINEAR AND NONLINEAR OSCILLATION THEORY OF DYNAMIC EQUATIONS ON TIME SCALES

Lynn Erbe, *Lincoln, USA*

We consider second order nonlinear and linear dynamic equations on a time scale and obtain a number of sufficient condidtions for oscillation or nonoscillation of solutions. The techniques used are variational, Riccati, or a combination of the two.

THE HARMONIC OSCILLATOR — AN EXTENSION VIA MEASURE CHAINS

Stefan Hilger, *Eichstätt, Germany*

In this talk I will present a discrete and a periodic version of the "harmonic oscillator equation". This means that the underlying group $\mathbb{R}$ of the classical harmonic oscillator will be replaced by the group $h\mathbb{Z}$ or by its dual group $\frac{1}{h}\,S$ (circle, radius $\frac{1}{h}$). Within this context we will examine the generalized versions of the following concepts surrounding the harmonic oscillator

- Annihilation and creation operators,
- Eigenvalues and eigenfunctions,
- Gauss function and Hermite polynomials,
- Fourier transform.

Also some related identities from mathematical physics, such as Heisenberg's uncertainty principle and the heat equation will be considered in this framework. It turns out that within the framework of calculus on measure chains we can closely track the effect of the $h \to 0$ transition in the above theory. One can also see that from the algebraic point of view the discrete and periodic theory is much richer and more interesting than the corresponding one for the continuous case.

AMS Classification: 39A12, 35Q40.

SOLUTION STABILITY OF SYSTEMS OF DIFFERENCE EQUATIONS WITH NONLINEARITY OF SPECIAL TYPE

Denys Khusainov, **I. V. Grizay**, *Kiev, Ukraine*

In this report nonlinear systems of difference equations with quadratic right hand size of the form

$$x(k+1) = x(k)X^T(k)Bx(k), \quad x(k) \in R^n, \quad k = 0, 1, 2 \tag{1}$$

in "supercritical case" are considered. Matrices

$$X^T(k) = \{X_1^T(k), x_2^T(k), \ldots, X_n^T(k)\}, B^T = \{B_1, B_@, \ldots, B_n\}$$

are $n^2 \times n$ square matrices. $X_i^T(k)$, $i = \overline{1,n}$ are matrices that have i-th row equal to the vector $x^T(k) = (x_1(k), x_2(k), \ldots, x_n(k))$ and other elements are zero. The following result was obtained.

Theorem 1. If elements of the matrices B_i, $i = \overline{1,n}$ satisfy conditions

$$b_{ii}^i = 0, \quad i = \overline{1,n}; 2b_{ij}^i + b_{ii}^j = 0, \quad i = \overline{1,n}, \quad j = \overline{1,n}, \quad i \neq j,$$
$$b_{ij}^k + b_{jk}^i + b_{ik}^j = 0, \quad i = \overline{1,n-2}, \quad j = \overline{1,n-2}, \quad k = \overline{1,n}, \quad i \neq j \neq k,$$

then zero solution of the system (1) is instable.

Systems of difference equations with delay are considered

$$x(k+1) = [I + \sum_{j=0}^{m} X^T(k-j)D_j]^{-1} \times \sum_{j=0}^{m} A_j x(k-j), \quad k = 0, 1, 2, \ldots \tag{2}$$

Theorem 2. Let constants $0 \leq \alpha_j \leq 1$, $j = \overline{1,m}$ exist and such a positively defined matrix H that the following formula is fulfilled

$$\frac{\lambda_{\min}(C)}{|A_0|^2|H|\varphi(H)}[1 - (\sum_{j=1}^{m} |A_j|)\varphi(H)]^2$$
$$> \sum_{j=1}^{m}(1+\alpha_j)|A_j|[2|A_0 + \sum_{s=1}^{m} \alpha_s A_s| + \sum_{s=1}^{m}(1+\alpha_s)|A_s|].$$

Then zero solution of the system (2) is asymptotically stable. The region of stability contains sphere S_R with radius

$$R = \frac{1}{\sqrt{\varphi(H)}} \min \left\{ z_0, \frac{1 - \sum_{j=1}^{m} |A_j|\sqrt{\varphi(H)}}{\sum_{j=1}^{m} |D_j|\sqrt{\varphi(H)}}, \frac{1}{|D_0| + \sum_{j=1}^{m} |D_j|\sqrt{\varphi(H)}} \right\},$$

z_0 is minimal square of cubical equation.

NONNEGATIVITY OF QUADRATIC FUNCTIONALS

Werner Kratz, *Ulm, Germany*

We consider Hamiltonian difference and differential systems, and we present new results (partly based on joined work with M. Bohner and O. Došlý), which characterize the nonnegativity of corresponding discrete and continuous quadratic functionals, respectively. It turns out that the required notion, the results and their proofs are very much the same for the discrete and the continuous case. The main tools for the proofs are generalized Picone formulae and the construction of particular examples.

OPEN PROBLEMS AND CONJECTURES

Gerry Ladas, *Kingston, USA*

We present some open problems and conjectures about some interesting classes of difference equations.

INTEGRABLE PARTIAL DIFFERENCE EQUATIONS; A MODERN PERSPECTIVE

Frank Nijhoff, *Leeds, United Kingdom*

Some recent developments in the theory of integrable partial difference equations will be reviewed. Such PΔEs form the precise analogues of the famous nonlinear evolution equations of soliton type such as the Korteweg-de-Vries equation. In recent years the insight in the structures behind such discrete equations, which share many of the miraculous properties with their differential counterparts, has advanced greatly. In the lecture I will explain the key integrability features of the partial difference equations, specific examples, their reductions and special solutions as well as their connection to certain nonlinear nonautonomous ordinary difference equations which are now considered to be the discrete analogues of the famous Painlevé transcendents (discrete Painlevé equations).

DISCRETE TIME DYNAMICS FROM OPTIMIZING MODEL IN ECONOMICS

Kazuo Nishimura, *Kyoto, Japan*

We will study the underlying structure of the two-dimensional dynamical system generated by a class of dynamic optimization models in economics. We establish global results, and relate them to the local analysis, by using the mathematical theory of two-dimensional dynamical systems.

LOCAL BIFURCATION OF PERIODIC SOLUTIONS OF FORCED EQUATIONS ON MANIFOLDS

Maria Patrizia Pera, *Florence, Italy*

We consider local bifurcation of forced oscillations of second order nonautonomous differential equations on differentiable manifolds.
A sufficient condition for an equilibrium point to be a bifurcation point of periodic orbits of the same period as that of the forcing term is obtained as an application of a quite general abstract finite dimensional result concerning bifurcation of fixed points from a suitable given manifold of trivial fixed points.

HIGHER ORDER DIFFERENCE EQUATIONS GENERATING A MONOTONE DISCRETE DYNAMICAL SYSTEM

Mihály Pituk, *Veszprem, Hungary*

Necessary and sufficient conditions are given under which a higher order difference equation generates a monotone discrete dynamical system with respect to the discrete exponential ordering. It is shown that under the above monotonicity assumption the stability properties (both local and global) of an equilibrium are equivalent to the stability properties of the same equilibrium of the associated first order equation. Applications to a second order equation from macroeconomics and to a model of haematopoiesis are given. The results were obtained jointly with Professor Ulrich Krause (Fachbereich Mathematik und Informatik, Universität Bremen, Germany).

GLOBAL STABILITY OF PERIODIC ORBITS OF NONAUTONOMOUS DIFFERENCE EQUATIONS AND POPULATION BIOLOGY

Robert J. Sacker, *Los Angeles, USA*
Saber Elaydi, *Texas, USA*

Elaydi and Yakubu showed that a globally asymptotically stable(GAS) periodic orbit in an autonomous difference equation must in fact be a fixed point whenever the phase space is connected. In this paper we extend this result to periodic nonautonomous difference equations via the concept of skew-product dynamical systems. We show that for a k-periodic difference equations, if a periodic orbit of period r is GAS, then r must be a divisor of k. In particular sub-harmonic, or long periodic, oscillations cannot occur. Moreover, if r divides k we construct a non-autonomous dynamical system having minimum period k and which has a GAS periodic orbit with minimum period r. Our methods are then applied to prove a conjecture by J. Cushing and S. Henson concerning a non-autonomous Beverton-Holt equation which arises in the study of the response of a population to a periodically fluctuating environmental force such as seasonal fluctuations in carrying capacity or demographic parameters like birth or death rates.

DYNAMICAL ISSUES ARISING FROM TIME DISCRETIZATIONS

George R. Sell, *Minneapolis, USA*

In this lecture we will examine various dynamical issues arising in the time discretizations of differential equations with time delays. Special emphasis will be placed on systems of such equations that have a Feedback Property. Our goal is to show that this is a rich area for future research into Difference Equations and Applications.

A FUNCTIONAL-ANALYTIC METHOD FOR THE STUDY OF DIFFERENCE EQUATIONS

Panayiotis Siafarikas, *Patras, Greece*

A functional-analytic method will be presented, for the study of existence and uniqueness of ordinary and partial, non-linear and linear difference equations in the Banach and Hilbert spaces of summable and square summable complex sequences. For non-linear difference equations a region, which depends on the initial conditions and the parameters of the equation where the solution holds, is given. Both for linear and non-linear equations, a bound of their solutions is determined. Applications will be given concerning difference equations arising from numerical schemes and physical or natural problems.

TRIANGULAR MAPS WITH ZERO TOPOLOGICAL ENTROPY - RECENT RESULTS AND OPEN PROBLEMS

Jaroslav Smítal, *Opava, Czech Republic*

For a continuous map of the interval there are several tens of equivalent conditions characterizing zero topological entropy. Most of these conditions are not equivalent for triangular maps $F(x, y) = (f(x), g(x, y))$ of the square, even when they are applicable. Based on very recent results obtained jointly with G.-L. Forti and L. Paganoni, we can exhibit the complete list of mutual relations between these conditions for triangular maps which are nondecreasing on the fibres. For the general triangular maps some relations still remain unknown, but again many open problems were recently solved by exhibiting some nontrivial examples. Regardless of this fact we can point out some interesting open problems along with conjectures concerning their solutions.

SYMMETRY PRESERVING DISCRETIZATIONS OF DIFFERENTIAL EQUATIONS AND THEIR APPLICATIONS AS NUMERICAL METHODS

Pavel Winternitz, *Montreal, Canada*

Different approaches to the study of continuous symmetries of difference equations will be presented. A related problem, that of creating difference schemes that have the same symmetry groups as the differential equations that they approximate, will be discussed. Numerical solutions of ordinary differential equations obtained using symmetry preserving schemes will be compared to solutions obtained by standard methods.

Short Communications

A NOTE ON REPRESENTATIONS OF SOLUTIONS OF EQUATIONS ON TIME SCALES

Ladislav Adamec, *Brno, Czech Republic*

We consider some possible representations of solutions of some very simple first order dynamic equations on time scale. By using these representations we investigate properties of solutions of some other dynamic systems on time scales.
This research was supported by the Czech Grant Agency under grant 201/01/0079.

SOME DYNAMIC INEQUALITIES ON TIME SCALES

Elvan Akın-Bohner, *Rolla, USA*

In the study of dynamic equations on time scales we deal with certain dynamic inequalities which provide explicit bounds on the unknown functions. These inequalities have been proved to be very useful in the analysis of various problems in the theory of dynamic equations.

QUASI-STATIC GROWTH OF BRITTLE FRACTURE: A VARIATIONAL METHODS BASED ON LOCAL MINIMIZATION

Samuel Babatunde Akinremi, *Serekunda, Nigeria*

The talk will deal with a variant of the variational model for the quasi-static growth of brittle fracture proposed by Francfort and Marigo.The main feature of this new model is that, in the discrete time formulation, in each step we do not consider absolute minimizers of energy, but, in a sense, we look for local minimizers which are sufficiently close to the approximately solution obtained in the previous step. This is done by introducing in the variation problem an additional term which penalizes the distance between the approximate solutiona at two consecutive times.The continuos-time version of this model is obtained by passing to the limit as the time step tends to zero.It satisfies (for almost every time)some minimality conditions which are slightly different from those considered in by Francfort and Marigo and by Toader but are still enough to prove (under suitable regularity assumptions on the crack path) that the classical Griffith's criterion holds at the crack tips.If no initial crack is present and if the data of the problem are sufficiently smoooth, no crack will develop in this new model, provided the penalization term is large enough.

ON INVERSES OF TRIDIAGONAL MATRICES

Qassem Al-Hassan, *Sharjan, United Arab Emirates*

An algorithm for computing the inverse of a general tridiagonal matrix is introduced. This algorithm is obtained by factoring this matrix a product of two bidiagonal matrices using Crout's LU factorization. A simple difference equation is used to generate a sequence of numbers, this sequence is then used to fill in the matrices L, U, L inverse, and U inverse, and consequently the required inverse.

SHARKOVSKII'S THEOREM AND DIFFERENTIAL EQUATIONS

Jan Andres, *Olomouc, Czech Republic*

Everybody knows the Sharkovskii cycle coexisting theorem. In spite of the apparent one-to-one correspondence between the periodic points of the related scalar Poincare translation operators and subharmonic solutions of the first-order (e.g. continuous) ordinary differential equations with the uniqueness property, the *classical Sharkovskii theorem cannot be applied to differential equations*. On the other hand, at the absence of uniqueness, the Poincare operators become multivalued. The *appropriate multivalued version will be therefore presented* at first. This can be expressed in terms of periodic orbits and it holds with the exception of at most two orbits. Then *it will be applied to differential equations* or, more generally, to scalar Caratheodory differential inclusions.

BOUNDED SOLUTIONS OF THIRD ORDER NONLINEAR DIFFERENCE EQUATIONS

Anna Andruch-Sobiło, **Małgorzata Migda**, *Poznań, Poland*

We consider a third order nonlinear difference equation

$$\Delta(a_n\Delta(b_n\Delta x_n)) = q_n f(x_{n+2}),\ n \in N,$$

where $\{a_n\}$, $\{b_n\}$, $\{q_n\}$ are positive real sequences, f is a real function with $xf(x) > 0$ for all $x \neq 0$.

We formulate the sufficient conditions for the boundedness of all nonoscillatory solutions of the above equation.

We consider also the problem of existence of quickly oscillatory solutions of the above equation.

ON SOME DISCRETE PROBLEMS RELATED TO DIFFERENTIAL-ALGEBRAIC EQUATIONS

Katalin Balla, *Budapest, Hungary*

When solving linear differential-algebraic equations, e.g. equations of the form $A(Dx)' + Bx = q$, with singular leading terms, several discrete problems arise. They may be of interest on their own. In the report, we intend to analyze some of them occurring in initial and boundary value problems.

INITIAL DATA GENERATING BOUNDED SOLUTIONS OF LINEAR DISCRETE EQUATIONS

Jaromír Baštinec, **Josef Diblík**, *Brno, Czech Republic*

Initial data generating solutions of linear discrete equations such that their graphs remain in a prescribed domain are found. For example the following holds: Let as consider the equation

$$\Delta u(k+1) = \varphi(k)u(k) + \delta(k),$$

and the initial problem

$$u(a) = u^*.$$

Let $b(k), c(k)$ are real function,$b(k) < c(k)$ for all $k \in N(a) = \{a, a+1, a+2, \dots\}$ and a is an integer. Let for every $k \in N(a), \varphi(k) > -1$ and the inequalities

$$(1+\varphi(k))b(k) + \delta(k) - b(k+1) < 0,$$

$$(1+\varphi(k))c(k) + \delta(k) - c(k+1) > 0,$$

hold. Then every initial problem with $u^* \in [b^*, c^*]$ determines a solution $u^*(k)$ satisfying inequalities

$$b(k) < u^*(k) < c(k)$$

for every $k \in N(a)$. Values b^*, c^* are the limits of special sequences.

Acknowledgment: This work was supported by the grant 201/01/0079 of Czech Grant Agency and by the Councils of Czech Government MSM 262200013 and MSM 262200022 of Czech Republic.

DICHOTOMY AND TRICHOTOMY ON THE WHOLE LINE Z FOR PERTURBED DIFFERENCE SYSTEMS

Alexander Boichuk, **Miroslava Růžičková**, *Žilina, Slovak Republic*

Bifurcation conditions of properties dichotomy and trichotomy on the whole line Z for difference systems

$$(L_\varepsilon x)(n) := x(n+1) - A(n)x(n) - \varepsilon A_1(n)x(n) = 0$$

are received under the assumption that the unperturbed system $(L_0 x)(n) = 0$ has a dichotomy on both half-lines Z_- and Z_+. Under suggestion conditions it is shown that $dim\ ker L_\varepsilon = r - d$ and $dim\ coker L_\varepsilon = 0$ where $r = dim\ ker L_0,\ d = dim\ coker L_0$; $(L_\varepsilon x)(n) : B(Z) \to B(Z)$ is the Banach space of vector valued function $x : Z \to R^N$ bounded on Z. Research supported by the grant VEG-A1/0026/03 of Slovak Grant Agency.

ON THE PERIOD FIVE TRICHOTOMY BEHAVIOR OF POSITIVE SOLUTIONS OF THE DIFFERENCE EQUATION $X_{N+1} = (P + X_{N-K})/X_N$

Elias Camouzis, *Athens, Greece*

We investigate the boundedness, global stability and periodic character of all positive solutions of the difference equation

$$x_{n+1} = \frac{p + x_{n-k}}{x_n},\ n = 0, 1, ...$$

where $k \in \{2, 3, ...\}$, p is a positive real number, and the initial conditions are arbitrary positive real numbers.

ON THE RELATED ASYMPTOTICS OF DELAY DIFFERENTIAL AND DIFFERENCE EQUATIONS

Jan Čermák, *Brno, Czech Republic*

We present some new asymptotic results for the linear delay differential equation

$$\dot{y}(t) = -a(t)y(t) + b(t)y(t-1)$$

with a positive continuous coefficient a and a nonzero continuous coefficient b. These results are based on a certain resemblance between the asymptotics of some delay differential equations and the asymptotics of the appropriate delay difference equations and inequalities.

SOME DISCRETE NONLINEAR INEQUALITIES AND APPLICATIONS TO BOUNDARY VALUE PROBLEMS

Wing-Sum Cheung, *Hong Kong*

We establish some new discrete Gronwall-Bellman-Ou-Iang type inequalities over 2-dimensional lattices. These on the one hand generalize some existing results in the literature and on the other hand, as they provide explicit bounds for the unknown functions, they provide a handy tool for the study of qualitative as well as quantitative properties of solutions of difference equations. We illustrate this by applying these new results to certain boundary value problem for difference equations.

PERIODIC SOLUTIONS OF NONLINEAR DIFFERENCE MATRIX EQUATIONS WITH BILINEAR MAIN PART

Vladimir Chiricalov, *Kyiv, Ukraine*

In this report we state and prove the main conditions for existence of periodic solutions of nonlinear difference periodic matrix equations with bilinear main part. We use the ideas of L.Cesari for finding periodic solutions of differential equation. The matrix solutions have been obtained as a limit of a uniformly convergent sequence of discrete periodic matrix-valued functions. The existence of periodic solutions of nonlinear matrix difference equations is related to existence of solutions of so-called *definding equation.* It solutions determine the initial point through which the solution of nonlinear equation passes at initial moment of time. As a simplest nonlinear matrix difference equations with bilinear main part may be considered the discrete-time matrix Riccati equation, which plays a crucial role in solving different tasks of the control system synthesis.

ON PERIODIC RATIONAL DIFFERENCE EQUATIONS OF ORDER K

Anna Cima, **Armengol Gasull**, **Francesc Mañosas** *Barcelona, Spain*

Consider the $k-$order rational difference equation

$$u_{n+k} = \frac{A_1 u_n + A_2 u_{n+1} + \cdots + A_k u_{n+k-1} + A_0}{B_1 u_n + B_2 u_{n+1} + \cdots + B_k u_{n+k-1} + B_0}, \tag{1}$$

with initial condition $(u_1, u_2, \ldots, u_k) \in (0, \infty)^k$, and such that $\sum_{i=0}^{k} A_i > 0$, $\sum_{i=0}^{k} B_i > 0$, $A_i \geq 0$, $B_i \geq 0$ and $A_1^2 + B_1^2 \neq 0$.
The talk is about the characterization of the difference equations (1) such that there exists $p \in \mathbb{N}$ for which $u_{n+p} = u_n$ for all $n \geq 1$. I'm going to explaine a first result that restricts the difference equations of type (1) that can be periodic for some $p \in \mathbb{N}$. As a consequence of this result, of the study of the periodic points associated to the dynamical system generated by (1) and of a powerful result of Csörnyei and Laczkovich, we can characterize all the difference equations (1) which are $p-$periodic for $k = 1, 2, 3, 4, 5, 7, 9$ and 11.

RETRACT PRINCIPLE AND POSITIVE SOLUTIONS OF DISCRETE EQUATIONS

Josef Diblík, *Brno, Czech Republic*

A powerful tool for investigation of various problems of ordinary differential equations as well as delayed differential equations is the retraction method. Recent extensions of the retract principle in the theory of discrete equations are discussed. The system of nonlinear discrete equations

$$\Delta u(k) = F(k, u(k))$$

with $F : N(a) \times R^n \to R^n$, $u = (u_1, \ldots, u_n)$, $k \in N(a) = \{a, a+1, a+2, \ldots\}$, $a \in \{0, 1, \ldots\}$ is considered.
Corresponding results concerning the existence of solutions with graphs lying in prescribed domains are formulated. The general principle is applied to concrete classes of difference equations. E.g. we derive sufficient conditions for existence of positive solutions of certain classes difference equations.
This research was supported by the Grant 201/01/0079 of the Grant Agency of Czech Republic and by the Plan of investigation MSM 2622 000 13 of the Czech Republic.

UNEXPECTED ISSUES FROM CLASSIC MATHEMATICAL MODEL OF POPULATION DYNAMICS

Vladimir Dobrynskiy, *Kiev, Ukraine*

We consider a light generalization of the simplest mathematical model for population dynamics describing of two insect species interaction and know as the "host and parasite" one.
We show there are parameter regions where the model has more than one (precisely, two) regimes of stable population coexistence.
So, we find a surprising issue : sometimes dynamics of the given classical model depends very essentially on initial numbers of insects in populations.

DISTRIBUTION OF ZEROS OF SOLUTIONS TO FUNCTIONAL EQUATIONS IN THE SPACE OF ESSENTIALLY BOUNDED FUNCTIONS

Alexander Domoshnitsky, *Ariel, Israel*

Definitions of zeros, oscillation and nonoscillation in case of noncontinuous solution are formulated. On this base oscillation and nonoscillation properties of solutions to functional equations are studied. Oscillation problems will be reduced to estimates of the spectral radius of corresponding operators acting in the space of essentially bounded functions. Various tests of oscillation and nonoscillation, generalizing known results for functional equations in the space of continuous functions and for difference equations, are obtained. Our main results are connected with estimates of a distance between zeros of solutions. Several assertions about zones of positivity of solutions to PDE are also obtained.

RECESSIVE AND DOMINANT SOLUTIONS FOR HALF-LINEAR DIFFERENCE EQUATIONS

Zuzana Došlá, *Brno, Czech Republic*

We present some recent results achieved in the joint research with M. Cecchi and M. Marini of University of Florence for the half-linear difference equation

$$\Delta(a_n\Phi(\Delta x_n)) = b_n\Phi(x_{n+1}),$$

where $\Phi_p(u) = |u|^{p-2}u$ with $p > 1$, $\{a_n\}$ and $\{b_n\}$ are positive real sequences for $n \geq 1$. Using the unique solvability of certain boundary value problems, recessive solutions are introduced as "smallest solutions in a neighbourhood of infinity". The equivalency with other properties, namely with the Riccati characterization and the convergence of a suitable series, will be given. The extension of the concept of the recessive solution to the quasilinear difference equations will be presented as well.

SHEARING TRANSFORMATIONS FOR SYMPLECTIC SYSTEMS

Julia Elyseeva, *Moscow, Russia*

We develop the technique of shearing transformations for symmetric matrices

$$\Lambda[V] = \operatorname{diag}\big[V_{21}^T \;\; V_{12}^T\big]V,\; \Omega[W] = V\operatorname{diag}\big[V_{12}^T \; V_{21}^T\big] = \Lambda\big[V^T\big],\; (1)$$

where $V = (V_{i\,j})$, $i, j = 1, 2$ are symplectic $2n \times 2n$-matrices. We apply our results to the oscillation theory of symplectic difference systems $Y_{i+1} = W_iY_i,\; i = 0, 1, \ldots, N,\;\; W_i^T JW_i = J$. In particular, we evaluate the number of focal points of a conjoined basis Y_i in $(i, i+1]$ using the index (the number of negative eigenvalues) of a matrix $\Lambda_j^*[V]$, where $\Lambda_j^*[V]$ is a shearing transformation for (1), and $V$is constructed for Y_i along a special integration path $j = j\,(i)$.

ON A CONDITION FOR TRANSITIVITY OF INTERVAL MAPS

Jose Luis Fachada, **João Ferreira Alves**, *Lisboa, Portugal*

Using kneading theory we introduce a topological invariant for interval maps. We show that the positivity of this invariant implies transitivity of the map.

SECOND SMALLER ZERO OF KNEADING DETERMINANT FOR ITERATED MAPS

Sara Fernandes, *Évora, Portugal*

(Joint work with J. Sousa Ramos.) The mixing rate on the studies of iterated maps is closed related with the second smaller zero of kneading determinant. For a certain class of maps (those with constant topological entropy) we use this zero to classify dynamical systems associated with difference equations.

ITERATED MULTIVALUED SYSTEMS

Jiří Fišer, *Olomouc, Czech Republic*

J. Hutchinson (1981) and M. Barnsley (1988) initiated the way to define and construct fractals as compact invariant subsets of a complete metric space X w.r.t. the union of contractive functions (iterated function systems—IFS)

$$\{f_i : X \to X, i = 1, \dots, n\}.$$

The proof is based on a simple application of the Banach contraction principle for Hutchinson-Barnsley operator

$$F : \mathcal{K}(X) \to \mathcal{K}(X), \quad F(A) = \bigcup_{x \in A} \bigcup_{i=1}^{n} f_i(x),$$

where $\mathcal{K}(X)$ denotes a hyperspace of nonempty compact subsets of X.
We present here our results concerning some further development consisting mainly in replacing the Banach contraction principle by some other fixed-point theorems, and in a multivalued generalization of IFS—iterated multifunction systems (IMS):

$$\{\varphi_i : X \to \mathcal{K}(X), i = 1, \dots, n\}.$$

where φ_i are multifunctions with nonempty compact values.

ON A NEW APPROACH TO THE LIMIT CYCLE PROBLEM

Valery A. Gaiko, *Minsk, Belarus*

We suggest a new global approach to solving Hilbert's sixteenth problem on the maximum number and relative position of limit cycles in two-dimensional quadratic systems. This approach can be applied also to arbitrary polynomial systems and to the global qualitative analysis of higher-dimensional dynamical systems. Namely: 1) to use five-parameter canonical systems with field-rotation (dynamic) parameters; 2) to divide the plane of two rest (static) parameters into the domains corresponding to various number and character of finite singularities and to consider the canonical systems separately in each of such domains, i. e. to reduce the study of limit cycle bifurcations to the analysis of three-parameter domains of dynamic parameters; 3) using the monotonicity of one-parameter families of multiple limit cycles generated by field-rotation parameters, to prove in every concrete case of finite singularities that the maximal one-parameter family of multiple limit cycles is not cyclic; 4) using Bautin's result on the cyclicity of a singular point which is equal to three and the Wintner–Perko termination principle stating that the multiplicity of limit cycles cannot be higher than the multiplicity (cyclicity) of the singular point in which they terminate, to prove by contradiction in every case the nonexistence neither of a multiplicity-four limit cycle nor of four limit cycles around a singular point; 5) to control simultaneously bifurcations of limit cycles around different singular points and to prove that the maximum number of limit cycles in a quadratic system is equal to four and the only possible their distribution is $(3:1)$.

ON THE TRICHOTOMY CHARACTER OF A DIFFERENCE EQUATION

Edward Grove, *Kingston, USA*

We study the global stability, the periodic character, and the boundedness nature of the difference equation

$$x_{n+1} = \frac{\alpha + \gamma x_{n-1}}{A + Bx_n + x_{n-2}}$$

where the parameters and the initial conditions are non-negative real numbers. We show that the solutions of the equation exhibit a trichotomy character which depends upon whether γ is less than A, equal to A, or greater than A.

THE ANALYTICAL AND NUMERICAL METHOD FOR SOME INVERSE HEAT TRANSFER PROBLEMS IN LAYERED MEDIA

Sharif Guseinov, *Riga, Latvia*

In the present work a simple method is proposed for reducing sufficiently wide classes of linear boundary value and initial boundary value direct and inverse problems of mathematical physics to the integral equations of the first kind Fredholm and Volterra type. Apart from that a two-dimensional non-stationary coefficient inverse problem of heat transfer is studied for two-layer media, where the coefficients of thermal diffusivity for both the media should be determined. It has been proved that the proposed method allows for finding the needed thermal diffusivity coefficients in an analytical form. Having analyzed the results obtained, the author makes a suggestion in the present work that the proposed method is universal enough to be applied when studying the following very important non-correct inverse problems of mathematical physics: 1. Retrospective inverse problems, in which the pre-history of the given state of the process is established. 2. Boundary inverse problems, where the boundary conditions or the related entering values are recovered. 3. Coefficient inverse problems, in which there are recovered some geometrical characteristics of the boundary surface or the coordinates of the measurement points of dependent variables inside this domain.

BUBBLE BIFURCATIONS IN THE ECONOMIC MODELS

Małgorzata Guzowska, *Szczecin, Poland*

We consider the problem of generating "bubble" bifurcations by economic and econometric models. We present the analytical and graphical analysis of the dynamic properties for models described by non-linear difference equations having two decision parameters.

BIFURCATION OF ALMOST PERIODIC SOLUTIONS IN DIFFERENCE EQUATIONS

Yoshihiro Hamaya, *Okayama, Japan*

Our subject in this paper is almost periodic solutions (in short, ap-solutions) originating from an equilibrium state when the parameters of the difference equation are varied. We consider the bifurcation of ap-solutions for an ap-difference equation with parameters of $x_{n+1} = f(n, x_n; \mu)$, using the Green's function for regular ap-operators and Σ-operators.

WHAT IS THE TRUE DISCRETE STRENGTHENED LEGENDRE CONDITION?

Roman Hilscher, *Brno, Czech Republic*

This is a joint work with Vera Zeidan (Michigan State University).
In the classical calculus of variations, the strengthened Legendre condition is an important hypothesis in the characterization of the nonnegativity of the second variation in terms of (i) the nonexistence of conjugate points and (ii) the solvability of the Riccati matrix differential equation. The parallel discrete-time notion, the "discrete strengthened Legendre condition", had been identified in the literature (including the authors themselves) as the positive definiteness of the diagonal block entries of a certain matrix, which represents the discrete-time second variation (discrete quadratic functional). However, the exact role of this "discrete strengthened Legendre condition" in the the theory of conjugate intervals (discrete-time notion parallel to conjugate points) was never explained. In this contribution, we shall explain the role of the mentioned "discrete strengthened Legendre condition" in discrete roundabout theorems and answer the question: *What is the true discrete strengthened Legendre condition in the discrete calculus of variations?* We shall also provide a brief explanation of the phenomenon through the analysis of the corresponding results on time scales.

DISCRETE CAUCHY-RIEMANN OPERATORS IN THE PLANE AS FOUNDATION OF AN APPROXIMATION OF NAVIER-STOKES AND VEKUA EQUATIONS

Angela Hommel, *Weimar, Germany*

A summary of the results based on discrete Cauchy-Riemann operators in the plane is presented. These operators allow a factorization of the real Laplacian into two adjoint Cauchy-Riemann operators. By the help of the discrete fundamental solution a right inverse operator to the discrete Cauchy-Riemann operator can be defined and a discrete Borel-Pompeiu formula is obtained. For a special case of the Navier-Stokes equations discrete potential and stream functions are calculated. The adapted model is related to the Cauchy-Riemann operator and can be immediately used for numerical calculations. Finally a discrete version of the Vekua equations is studied. It is proved in a special case that any solution of the homogeneous difference equation can be written as a product of two factors where one factor is a discrete holomorph function.

MOMENTS OF SOLUTIONS TO A LINEAR DIFFERENCE EQUATION

Klara Janglajew, *Bialystok, Poland*

A linear nonhomogeneous difference equation with coefficient dependent on two-neighboring values of Markov chain is studied. For this equation we shall derive moment equations. A linear system of this kind difference equations is also considered.

DISCRETE PICONE'S IDENTITY AND ITS APPLICATIONS IN THE COMPARISON THEORY OF NONLINEAR DIFFERENCE EQUATIONS

Jaroslav Jaroš, *Bratislava, Slovakia*

The purpose of the talk is to present new comparison results of the Sturm-type obtained with the help of a discrete version of Picone's identity. It is shown that the oscillatory behaviour of solutions for a class of nonlinear difference equations can be deduced from the oscillation of certain associated linear (or half-linear) difference equation.

OSCILLATION AND NONOSCILLATION FOR TWO-TERMS DIFFERENCE EQUATIONS OF THE THIRD ORDER

Aleš Kobza, *Brno, Czech Republic*

The third order nonlinear difference equations

$$\Delta(p_n\Delta(r_n\Delta x_n)) - q_n f(x_{n+p}) = 0, \quad p \in \{0,1,2\}, \tag{E$_p$}$$

where (p_n), (r_n) and (q_n) are sequences of positive real numbers for $n \in \mathbb{N}$, $f : \mathbb{R} \to \mathbb{R}$ is a continuous function such that $f(u)u > 0$ for $u \neq 0$ are investigated. A special attention is paid to equation (E_2) for which the generalized zeros of solutions are studied and an energy function G is introduced. The relation between the oscillatory solutions and solutions for which $G_n > 0$ for $n \in \mathbb{N}$ is established.

OPINION DYNAMICS IN HIGHER DIMENSIONS

Ulrich Krause, *Bremen, Germany*

The talk treats the *formation of opinions* among n people (experts, agents, ...) where opinions are described by collections of d real data. If $x^i(t)$ denotes the opinion of agent i at time step $t \in \mathbb{N} = \{0,1,2,\ldots\}$ then the *opinion dynamics* is given by $x^i(t+1) = M_i(x^1(t),\ldots,x^n(t))$, where $x^i(t) \in S^i \subset \mathbb{R}^d_+, S^i$ the state space of agent i and $M_i \colon S_i^n \to S_i$ is a *mean procedure* by which agent i takes the opinions of the other agents into account. The talk will focus on the question under which conditions the agents are able to reach a *consensus*, that is $\lim\limits_{t\to\infty} x^i(t) = c$ for *all* i. Applications range from sociology over economics to physics and mathematics. A simple but not trivial special case of opinion dynamics is the well–known *arithmetic–geometric mean of Gauss*. In this example $n = 2, S_1 = S_2 = \mathbb{R}_+$, M_1 = arithmetic mean, M_2 = geometric mean. The agents reach a consensus which, surprisingly, is given by an elliptic integral. More general, consensus formation may be analyzed for a *Gauss soup* where an arbitrary number of agents apply an (weighted) arithmetic or geometric mean.

A QUASI-LINEARIZATION TECHNIQUE FOR A DYNAMIC INITIAL VALUE PROBLEM ON TIME SCALES AND THOUGHTS BEYOND

Bonita Lawrence, *West Virginia, USA*

In the spotlight of this study is a particular type of dynamic initial value problem of the form
$u^{\Delta} = f(t,u) + g(t,u)$
$u(t_0) = u_0$
where f, g in $C_{rd}[T^k R, R]$ are non-decreasing and non-increasing, respectively. A quasi-linearization technique utilizing the nature of natural lower and upper solutions as well as coupled lower and upper solutions is developed for this problem. Beginning with the existence of coupled lower and upper solutions we create two sequences that converge to minimal and maximal solutions. Ultimately, the goal is determine when the sequences converge to a unique solution of the problem. Some thoughts on special cases are also presented.

EXISTENCE OF POSITIVE PERIODIC SOLUTIONS OF A CLASS OF DELAY DIFFERENCE EQUATIONS AND APPLICATIONS IN POPULATION DYNAMICS

Wan-Tong Li, *Gansu, P. R. China*

With the help of continuation theorem based on Mawhin's coincidence degree, the existence of a positive periodic solution of the delay difference equation

$$x(k+1) = x(k) \exp\{F(k, x(k-\tau_1), ..., x(k-\tau_n))\}$$

are studied. When these results are applied to some special delay population models, some new results are obtained.

Key Words: Delay difference equation, positive periodic solution, Fredholm mapping, Mawhin coincidence degree theory.
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POINTWISE MULTIPLICATION IN GENERALIZED BESOV AND LIZORKIN-TRIEBEL SPACES

Moussai Madani, *M'Sila, Algeria*

The product $A^{s,q_1}_{p_1} \cdot A^{r,q_2}_{p_2}$ can be included in $A^{s,q}_{p}$ under some hypotheses on $p, p_1, p_2, q, q_1, q_2, r$ and s where A with the 3 indices will define either a F-space of Lizorkin-Triebel or a B-space of Besov. We prove the sufficient conditions on above parameters such that the embeddings of type $F \cdot B \hookrightarrow F$, $F \cdot (F \cap L^\infty) \hookrightarrow F$ and $B \cdot (B \cap L^\infty) \hookrightarrow B$ hold.

ON A CLASS OF FOURTH ORDER NONLINEAR DIFFERENCE EQUATIONS

Małgorzata Migda, **Anna Musielak**, **Ewa Schmeidel** *Poznań, Poland*

We consider the fourth order nonlinear difference equation

$$\Delta(a_n \Delta(b_n \Delta(c_n \Delta y_n))) + f(n, y_n) = 0, \ n \in N$$

where $a, b, c : N \to R_+$ and the series $\sum\limits_{n=1}^{\infty} \frac{1}{a_n}$, $\sum\limits_{n=1}^{\infty} \frac{1}{b_n}$, $\sum\limits_{n=1}^{\infty} \frac{1}{c_n}$ are convergent, $f : N \times R \to R$ is a continuous function with $xf(n, x) > 0$ for $x \neq 0$. We obtain necessary and sufficient conditions for the existence of nonoscillatory solutions of the above equation, with special asymptotic behavior.

ITERATES OF THE TANGENT MAP - THE BIFURCATION SCHEME

Henrique Oliveira, *Lisbon, Portugal*

In this work we study the bifurcations of a real non continuous map, related to the complex tangent map $\lambda \tan z$.

We present conditions for various types of bifurcations. We discuss also the structure of the sequence of bifurcation which occur in this type of maps.

ON ONE ASYMPTOTIC FORMULA FOR THE FINITE DIFFERENCE EQUATIONS

Andrey Osipov, *Tahtelantie, Finland*

We consider the following infinite system of finite difference equations

$$\mu_n u_{n-q} + a_{n,n-q+1}u_{n-q+1} + \cdots + a_{n,n+p-1}u_{n+p-1} + \lambda_n u_{n+p} = \lambda y_n, \qquad (*)$$

$$n > q, \quad \mu_n \neq 0, \quad \lambda_n \neq 0,$$

(for some fixed indices $q, p \geq 1$ and parameter $\lambda \in \mathbb{C}$) with the complex coefficients,satisfying the following conditions:

$$\mu_n = n^\alpha(1+\xi_n),\ \lambda_n = n^\alpha(1+\delta_n);\ \{\delta_n\},\{\xi_n\} \in l^2,\ \alpha \in (1/2,\infty)$$
$$\sum_{n=q+1}^{\infty} |a_{n,n-i}|n^{-\alpha} < \infty,\ i = -q+1,\ldots,p-1.$$

Under some additional requirements on $\{\delta_n\},\{\xi_n\}$ we obtain the asymptotic formula for the solutions of (*) similar to the formula used in the spectral analysis of discrete Schrödinger operator.We apply the obtained formula to prove, that the analog of the Hellinger-Wall theorem of invariance in l^2, is not valid in l^p for $p > 2$.

A LIMIT SET TRICHOTOMY FOR SYSTEMS ON TIME SCALES

Christian Pötzsche, *Augsburg, Germany*

In this talk we present a limit set trichotomy for abstract order-preserving 2-parameter semiflows in normal cones of strongly ordered Banach spaces. Additionally, to provide an example, Müller's theorem is generalized to dynamic equations on arbitrary time scales and applied to a model from population dynamics. Keywords: Limit set trichotomy, 2-parameter semiflow, dynamic equation, time scale; AMS Classification: Primary 37C65; Secondary 37B55, 92D25 (joint-work with Stefan Siegmund)

TRICHOTOMY OF A SYSTEM OF TWO DIFFERENCE EQUATIONS

Garyfalos Papaschinopoulos, **Gesthimani Stefanidou**, *Xanthi, Greece*

We study the boundedness and the asymptotic behavior of the positive solutions of the system of difference equations

$$x_{n+1} = A + \frac{\sum_{i=1}^{k} a_i x_{n-p_i}}{\sum_{j=1}^{m} b_j y_{n-q_j}}, \quad y_{n+1} = B + \frac{\sum_{i=1}^{k} c_i y_{n-p_i}}{\sum_{j=1}^{m} d_j x_{n-q_j}},$$

where $k, m \in \{1, 2, ...\}$, A, B, a_i, c_i, b_j, d_j, $i \in \{1, ..., k\}$, $j \in \{1, ..., m\}$ are positive constants, p_i, q_j, $i \in \{1, ..., k\}$, $j \in \{1, ..., m\}$ are positive integers such that $p_1 < p_2 < ... < p_k$, $q_1 < q_2 < ... < q_m$ and the initial values x_i, y_i, $i \in \{-\pi, -\pi + 1, ..., 0\}$, $\pi = \max\{p_k, q_m\}$ are positive numbers.

DIFFERENCE EQUATIONS FOR PHOTON NUMBER DISTRIBUTION IN THE STATIONARY REGIME OF A RANDOM LASER

Vlasta Peřinová, *Olomouc, Czech Republic*

The equations of population dynamics can be applied in many fields. They appear in the model for a random laser, but completed with Langevin terms, which corresponds to a stochastic dynamics. As usual, the joint probability distribution $p(n_1, ..., n_{N_p}, N_1, ..., N_{N_s}, t)$ of photon numbers and of densities of excited atoms obey rate equations, i.e., differential–difference equations and an initial condition at $t = t_0$. The steady-state probabilities of the process, i.e., those for $t_0 \to -\infty$, obey difference equations.

SOLUTIONS IN ℓ_2 OF LINEAR SYSTEMS OF DIFFERENCE EQUATIONS

Eugenia Petropoulou, *Patras, Greece*

We give sufficient conditions for the existence and the uniqueness of complex ℓ_2 solutions of a general class of non-delay system of linear difference equations and of two general classes of delay systems of linear difference equations. In some cases bounds of the established solutions are also given. As a consequence of the space ℓ_2 where we work, information can be obtained about the asymptotic behavior of the established solutions and, the asymptotic stability of the zero equilibrium point of the systems under consideration. The method we use is a functional-analytic one.

ON THE COMPATIBILITY CONDITIONS AND ON THE MANIFOLDS OF THE SOLUTIONS OF THE QUASI-LINEAR EQUATION SYSTEMS OF THE FOUR DIFFERENTIAL EQUATIONS WITH THE THREE UNKNOWN FUNCTIONS IN THE SPACE.

Rakhmon Pirov, *Dushanbe, Tajikistan*

The report includes the investigation of two types of quasi-linear systems of four differential equations in third-dimensional space (there is no doubts in relevance of studying these systems cause many tasks in hydro and gas dynamics, the theory of elasticity, the theory of field are narrowed down to the given tasks). The vivid conditions of compatibility are found, providing the single-valued solutions of different tasks with inceptive data. More exact, it is proved that on the acquittal of compatibility conditions of the manifolds of the solutions of investigated systems contain: - two random constants - two random constants and one random function of one variable - three random functions of one variable The main method of stydying is in narrowing down of studied systems to the systems with full differentials.

THE ABEL DYNAMIC EQUATION

Zdeněk Pospíšil, *Brno, Czech Republic*

The dynamic equation

$$x^{\Delta}(t) = \frac{a(t)x(t)^3 + b(t)x(t)^2 + c(t)x(t)}{1 - \mu(t)(a(t)x(t)^2 + b(t)x(t))}$$

on a discrete time scale $\mathbb{T} = \{t_n\}_{n=0}^{\infty}$ is considered. Under some assumptions, this equation can be transformed onto "quasi Euler-Cauchy" dynamic equation

$$\tau\sigma(\tau)\xi^{\Delta\Delta}(\tau) + g\big(\xi(\tau)\big) = 0$$

on another discrete time scale.
An application — a model of population exhibiting both intraspecific competition and cooperation — is also discussed.

DISCRETE-TIME HAMILTONIAN SYSTEMS: λ-ZONES AND PARAMETRIC RESONANCE

Vladimir Răsvan, *Craiova, Romania*

The motivation of this paper is twofold. First it starts from the idea that the theory of λ-zones developed by M. G. Krein and his followers has a discrete-time counterpart. Within this framework several results have been obtained and published. The newly emerging theory of systems on time scales is also stimulating this type of research. On the other hand the theory of λ-zones is the best framework for parametric resonance studies. For this reason the next step of the programme concerning discrete-time Hamiltonian systems and their λ-zones of stability would be to define a proper model for parametric resonance and to extend the basic results on critical frequencies. in this paper this step is sketched based on the same early results of Krein (1955) and also on the basic reference by Yakubovich and Staržinskii (Parametric resonance in linear systems, 1987).

A HARDY INEQUALITY AND HALF-LINEAR DYNAMIC EQUATIONS ON TIME SCALES

Pavel Řehák, *Brno, Czech Republic*

A Hardy inequality on time scales will be presented, which unifies and extends well-known integral and sum Hardy inequalities. As an application we will examine oscillatory behavior of a generalized (half-linear) Euler dynamic equation. The results turn out to be new even in the particular linear case. The questions how the graininess of the time scale affects oscillation/nonoscillation of the equation will be discussed. In particular, we look for the "right" form of the equation where a time scale-invariant critical constant exists, and we give an example of the equation which becomes oscillatory (although, it was originally nonoscillatory) when replacing certain time scale by a different one with a "large" graininess.

NONLINEAR BOUNDARY VALUE PROBLEMS ON SEQUENCE SPACES

Jesus Rodriguez, *North Carolina, USA*

In this paper we study nonlinear, discrete-time systems subject to global boundary conditions. The problems considered are of the form

$$x(k+1) = f(\lambda, k, x(k)); \quad k = 0, 1, 2, 3, \cdots$$

subject to

$$G(\lambda, x) = 0.$$

In these problems λ is a real parameter, f and G are smooth mapping, $f : \mathbb{R}^{n+2} \to \mathbb{R}^n$, and $G : \mathbb{R} \times l_\infty \to \mathbb{R}^p$ where $p \leq n$.
We establish sufficient conditions for the existence of l_∞ solutions and we analyze the extent to which such problems remain solvable under perturbations in the parameter λ. Connections will be presented between these constrained dynamical systems and boundary value problems in differential equations.

ANALYTIC PROPERTIES OF A SPECIAL Q-EXPONENTIAL FUNCTION

Andreas Ruffing, *Garching, Germany*

$$(\boldsymbol{\Delta_q} f)(z) := \tfrac{f(qz)-f(q^{-1}z)}{qz-q^{-1}z} \qquad \forall\ z \in \mathbb{C} \setminus \{0\}$$

yields for $q \in (0,1)$ the so-called symmetric q-difference operator on $\mathbb{C} \setminus \{0\}$. Holomorphic solutions to the fixed point problem of this q-difference operator and of $\boldsymbol{\Delta_q}^2$ are elaborated. The analytic properties of the corresponding holomorphic functions are investigated. A link between the eigenfunctions and the q-Fourier transform by Koornwinder and Swarttouw on a q-linear grid is established. The meaning of the fixed point problem to discrete Schrödinger theory is mentioned.

ON THE SYSTEM OF TWO DIFERENCE EQUATIONS $X_{N+1} = (P + Y_{N-2})/Y_N$, $Y_{N+1} = (Q + X_{N-2})/X_N$

Christos Schinas, **Garyfalos Papaschinopoulos**, *Xanthi, Greece*

We investigate the boundedness, the periodicity and the asymptotic behavior of the positive solutions of the system of difference equations

$$x_{n+1} = \frac{p + y_{n-2}}{y_n}, \quad y_{n+1} = \frac{q + x_{n-2}}{x_n}, \quad n = 0, 1, ...,$$

where p, q are positive constants and the initial values x_i, y_i, $i \in \{-2, -1, 0\}$ are positive numbers.

ASYMPTOTIC PROPERTIES OF FOURTH ORDER NONLINEAR DIFFERENCE EQUATIONS WITH QUASIDIFFERENCES

Ewa Schmeidel, **Małgorzata Migda**, *Poznań, Poland*

We consider fourth order nonlinear difference equation of the form

$$\Delta(a_n\Delta(b_n\Delta(c_n\Delta y_n))) + f(n, y_n) = 0,\ n \in N$$

where $a, b, c : N \to R_+$ and a_n is bounded away from zero, $a_n \geq c_n$, the series $\sum\limits_{i=1}^{\infty} \frac{1}{a_i}$ is convergent, the series $\sum\limits_{i=1}^{\infty} \frac{1}{b_i}$ is divergent and $\sum\limits_{j=1}^{\infty} \frac{1}{c_j} \sum\limits_{i=j}^{\infty} \frac{1}{b_i} < \infty$. The function $f : N \times R \to R$ is continuous and monotonic on second argument and $xf(n, x) > 0$ for $x \neq 0$. The classification of nonoscillatory solutions are given. Sufficient conditions are obtained for the difference equation to admit the existence of nonoscillatory solutions with special asymptotic properties.

THE BANACH SPACE OF POSITIVELY REGRESSIVE FUNCTIONS ON A TIME SCALE

Moritz Simon, *Garching, Germany*

It is a well-known fact in the calculus on time scales that the set $\mathcal{R}^+$ of all positively regressive (and rd-continuous) functions on a time scale $\mathbb{T}$ supplied with the (circle plus) addition $\oplus$ and the (circle dot) scalar multiplication $\odot$ is a real vector space (see e.g. chapter 2.5 in "Advances in Dynamic Equations on Time Scales" edited by Martin Bohner and Allan Peterson). We shall introduce a norm $||\,.\,||_\mu$ for this space of functions which degenerates to the supremum norm $||\,.\,||_\infty$ in the continuum case $\mu = 0$. The set $\mathcal{B}^+$ of all $\mathcal{R}^+$-functions which are bounded with respect to this norm then becomes a normed space $(\mathcal{B}^+, ||\,.\,||_\mu)$. Our main goal will be to prove that this normed space is complete, i.e. a Banach space. For a bounded time scale $\mathbb{T}$ we have $\mathcal{B}^+ = \mathcal{R}^+$. Especially, $(\mathcal{R}^+, ||\,.\,||_\mu)$ coincides with the already known Banach space $(\mathcal{C}[a, b], ||\,.\,||_\infty)$ of continuous functions in the case $\mu = 0$. However, we shall also consider discrete examples in order to underline the characteristic properties of $||\,.\,||_\mu$.

ASYMPTOTIC BEHAVIOR OF THE SOLUTIONS OF A FUZZY DIFFERENCE EQUATION

Gesthimani Stefanidou, **Garyfalos Papaschinopoulos**, *Xanthi, Greece*

We study the existence, the uniqueness, the boundedness and the asymptotic behavior of the positive solutions of the following fuzzy difference equation:

$$x_{n+1} = \frac{A + \sum_{i=0}^{k} a_i x_{n-i}}{B + \sum_{i=0}^{k} b_i x_{n-i}}$$

where $k \in \{1, 2, ..., \}$, A, B are positive fuzzy numbers and a_i, b_i, $i \in \{0, 1, ..., k\}$ are positive fuzzy numbers.

ON THE GAUSS HYPERGEOMETRIC SERIES WITH ROOTS OUTSIDE THE UNIT DISK

Katsuo Takano, **H. Okazaki**, *Ibaraki, Japan*

It is known that the normed conjugate product of gamma functions such as $\frac{2}{\pi}|\Gamma(1+ix)|^2$ is an infinitely divisible density and in the process in showing the infinite divisibility of this probability distribution a family of polynomials with roots outside the unit disk appeared. In this talk the speaker will talk about the location of roots of the Gauss hypergeometric series ${}_2F_1(2m, -n; 2m+n+1; z)$.

THE 3D HAPPEL MODEL FOR COMPLETE ISOTROPIC STOKES FLOW

Panayiotis Vafeas, *Patras, Greece*

Particleincell models for the steady and nonaxisymmetric flow of incompressible, viscous fluids at low Reynolds number (Stokes flow) are useful in the development of simple but reliable analytical expressions for swarms of particles. It is of great theoretical and practical interest the investigation of threedimensional flow in assemblages of such particles. Here the creeping flow through a swarm of spherical particles that translate and rotate in a quiescent Newtonian fluid, is analyzed with a 3D sphereincell model. The mathematical treatment is based on the two concentric spheres model. The inner sphere comprises one of the particles in the swarm and the outer sphere consists of a fluid envelope. The appropriate boundary conditions are applied, namely, nonslip flow condition on the surface of the solid sphere and nil normal velocity component and shear stress on the external spherical surface. The boundary value problem is solved with the aim of the Papkovich Neuber differential representation of the solutions for Stokes flow, which is valid in nonaxisymmetric geometries and provides us with the velocity and total pressure fields in terms of harmonic spherical eigenfunctions. The solution is obtained in closed form and analytical expressions for the velocity, the total pressure, the angular velocity and the stress tensor fields are provided.

SYMBOLIC DYNAMICS IN THE TIME-DELAYED CHUA'S CIRCUIT

Sandra Vinagre, *Évora, Portugal*

In this paper we consider the time-delayed Chua's circuit introduced by Sharkovsky (IJBC 94) whose behavior is determined by properties of a one-dimensional map. We study this map in terms of symbolic dynamics which makes possible to characterize the associated time evolution of the time-delayed Chua's circuit.

STABILITY OF IMPULSIVE FUNCTIONAL DIFFERENTIAL EQUATIONS WITH JUMPS AT STOPPING TIMES

Shujin Wu, *Shanghai, China*

The paper deals with the investigation of the uniform stability of the zero solution of nonlinear impulsive functional differential equations with jumps at stopping times. First, the model of impulsive functional differential systems with jumps at stopping times are brought forward. Then, by means of Liapunov functions coupled with Razumikhin technique sufficient conditions for uniform $p-$moment stability and uniform strong stability of the zero solution to such equations are present. At last, our results are applied to the population dynamics to show their applications.

COEXISTENCE PROBLEMS FOR THE HILL EQUATIONS WITH 3-STEP POTENTIALS

Kazushi Yoshitomi, *Tokyo, Japan*

We study the coexistence of two linearly independent, periodic solutions of the Hill equation with a 3-step potential. We state a simple necessary and sufficient condition for the coexistence. Using this condition, we provide a formula for the number of joints of the Arnold tongue of a family of the Hill equations with 3-step potentials.

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