



*O.D. Equations Brno 2016*

# Program and Abstracts

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*The financial support received from*

*Department of Mathematics and Statistics, Faculty of Science,  
Masaryk University*

*is highly appreciated.*



# *O.D. Equations Brno 2016*

Dedicated to Professor Ondřej Došlý  
on the occasion of his 60<sup>th</sup> birthday

June 6–8, 2010, Brno, Czech Republic

*organized by*

**Department of Mathematics and Statistics, Faculty of Science,  
Masaryk University**

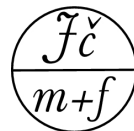
**Institute of Mathematics,  
Czech Academy of Sciences**

*in cooperation with*

**Union of Czech Mathematicians and Physicists,  
Brno branch**



**The Czech Academy  
of Sciences**





# Preface

The conference O.D.Equations Brno 2016 (also in short ODE 60) is organized on the occasion of the 60th birthday of Professor Ondřej Došlý, a distinguished professor of mathematical analysis at the Department of Mathematics and Statistics of the Faculty of Science, Masaryk University, our dear colleague, teacher, and friend. At the same time we would like to celebrate 60 years from the publication of one of the first important papers on ODEs by Professor Otakar Borůvka, who is regarded as the founder the Brno school of differential equations: *Sur la transformation des intégrales des équations différentielles linéaires ordinaires du second ordre*, Annali di Matematica Pura ed Applicata 41, 325–342 (1956).

The program of the conference consists of plenary and invited lectures and short communications. The topics cover the theory of differential equations and difference equations in a broad sense. The social program consists of a visit to Villa Tugendhat, placed on UNESCO List of World Cultural Heritage, and a conference dinner at Špilberk Castle in Brno. Over sixty registered participants arrive at the conference from ten European countries, USA, and Japan. We hope that all will enjoy the scientific and social program of the conference, our beautiful campus, and the city of Brno.

The organizers of the conference are grateful for support and constant help of many colleagues and students, in particular, to Michaela Burešová, Eva Dobrovolná, Vladimíra Chudáčková, Milada Suchomelová, Petr Zemánek, Jana Zuzáková. We would like express our very special thanks to Zuzana Došlá and Petr Liška.

Dear Ondřej and Brno school of differential equations, we wish you happy birthday. We are proud of you.

Brno, June 2016

Pavel Řehák and Roman Šimon Hilscher  
The organizers



# Program

## Monday, 6th June

12:45 – 13:00 **Opening Session**

### Plenary Lecture

13:00 – 14:00 **Werner Kratz:** Oscillation and spectral theory of continuous Hamiltonian and discrete symplectic systems with applications

### Invited Lectures

14:00 – 14:30 **Pavel Drábek:** Oscillation and nonoscillation results for half-linear equations with deviated argument

14:30 – 15:00 **Julia Elyseeva:** Relative oscillation of linear Hamiltonian differential systems without controllability

### Coffee Break

### Invited Lecture

15:30 – 16:00 **Mehmet Ünal:** Linearization techniques of half-linear differential equations

### Short Communications

16:00 – 16:15 **Pavel Řehák:** Discrete world and half-linear world

16:15 – 16:30 **Vladimir Rasvan:** On some models occurring in synchronization

16:30 – 16:45 **Simona Fišnarová:** Modified Riccati technique for half-linear equations: ordinary and delay case

16:45 – 17:00 **Roman Šimon Hilscher:** Symplectic systems in oscillation, spectral and variational theory

### Coffee Break

### Short Communications

17:30 – 17:45 **Mihály Pituk:** Linearized oscillation for nonautonomous delay differential equations

17:45 – 18:00 **Abdullah Özbekler:** Disconjugacy via Lyapunov and Vallée-Poussin type inequalities for forced differential equations

18:00 – 18:15 **Tibor Krisztin:** Differential equations with queueing delays

18:15 – 18:30 **Ewa Schmeidel:** Stability in demand-inventory model

## Tuesday, 7th June

### Plenary Lecture

9:00 – 10:00 **Martin Bohner:** Periodicity in the Beverton–Holt population model applications

### Invited Lectures

10:00 – 10:30 **Jean Mawhin:** First order difference systems with multipoint boundary conditions

10:30 – 11:00 **Stefan Hilger:**  $sl_2$  and friends

### Coffee Break

### Invited Lectures

11:30 – 12:00 **Jaroslav Jaroš:** Picone's identity revisited

12:00 – 12:30 **Gabriella Bognár:** On the solutions to growth equations and surface pattern models

### Short Communications

12:30 – 12:45 **Petr Stehlík:** Well-posedness and maximum principles for lattice reaction-diffusion equations

12:45 – 13:00 **Nahed A. Mohamady:** Boundedness of positive solutions of a system of nonlinear delay differential equations

### Trip to Villa Tugendhat

15:00 – 16:30 **First Group** We will leave at 14:15 from hotel Continental. We will use public transportation, tickets will be provided.

16:00 – 17:30 **Second Group** We will leave at 15:15 from hotel Continental. We will use public transportation, tickets will be provided.

### Conference Dinner

19:00 – **Špilberk Castle** We will leave at 18:30 from hotel Continental, 15 minutes walk.



## Wednesday, 8th June

### Plenary Lecture

9:00 – 10:00 **Naoto Yamaoka:** Oscillation criteria for second-order nonlinear differential equations with  $p$ -Laplacian

### Invited Lectures

10:00 – 10:30 **John R. Graef:** Multiple Anti-Periodic Solutions to Nonlinear Fourth Order Difference Equations

10:30 – 11:00 **Alexander Lomtatidze:** On zeros of solutions of a certain nonlinear ordinary differential equation

### Coffee Break

### Invited Lecture

11:30 – 12:00 **Robert Mařík:** ODE methods outside the ODE world

### Short Communications

12:00 – 12:15 **Mirosława Zima:** Existence results for the boundary value problem related to the Liebau phenomenon

12:15 – 12:30 **Feliz Minhós:** Third-order couple systems with dependence on the first derivative

12:30 – 12:45 **Jonáš Volek:** Landesman-Lazer conditions for difference equations involving sublinear perturbations

12:45 – 13:00 **Irina Astashova:** On oscillation of solutions to higher-order equations with power nonlinearity

13:00 – 13:15 **Giselle A. Monteiro:** Extremal solutions of measure differential equations and application to impulsive systems

13:15 – 13:30 **Milan Medved':** Exponential stability of solutions of some multi-fractional differential equations



# ON OSCILLATION OF SOLUTIONS TO HIGHER-ORDER EQUATIONS WITH POWER NONLINEARITY

Irina Astashova, *Moscow, Russia*

For the equation

$$y^{(n)} + P(x, y, y', \dots, y^{(n-1)}) |y|^k \operatorname{sgn} y = 0, \quad n \geq 2, k \in (0, 1) \cup (1, \infty) \quad (1)$$

with nonzero continuous function  $P$  and its special cases

$$y^{(n)} + \sum_{i=0}^{n-1} a_j(x) y^{(i)} + p(x) |y|^k \operatorname{sgn} y = 0, \quad k > 1 \quad (2)$$

with continuous functions  $p, a_j, j = 0, \dots, n-1$ , and

$$y^{(n)} + p_0 |y|^k \operatorname{sgn} y = 0, \quad n > 2, k \in (0, 1) \cup (1, \infty), p_0 \in \mathbb{R} \setminus 0, \quad (3)$$

properties of oscillatory solutions are described. For equation (1) some results are obtained on the existence of oscillatory solutions in the case  $n > 2$  [1], [2] and on their asymptotic behavior in the case of  $n = 3, 4$  [2]. For equation (2) a criterion is obtained for oscillation of all its solutions if  $n$  is even and  $\int_x^\infty x^{n-j-1} |a_j(x)| dx < \infty, j = 0, \dots, n-1$  [3]. For equation (3) with  $n = 4$  the existence of periodic solutions is proved. The existence of a special type of oscillatory solutions is proved as in the following theorem.

**Theorem 1.** *For any integer  $n > 2$  and any  $k \in (0, 1) \cup (1, \infty)$  there exists a non-constant oscillatory periodic function  $h(s)$  such that for any  $x^* \in \mathbb{R}$  and any  $p_0$  satisfying  $(-1)^n p_0 > 0$  the function*

$$y(x) = |p_0|^{\frac{1}{1-k}} (x - x^*)^{-\alpha} h(\log(x - x^*)), \quad x > x^*, \quad (4)$$

*is a solution to equation (3).*

- [1] Kiguradze I.T., Chanturia T.A. Asymptotic properties of solutions of nonautonomous ordinary differential equations, Kluwer Acad. Publ. G, Dordrecht, 1993.
- [2] Astashova, I. V. Qualitative properties of solutions to quasilinear ordinary differential equations. In: Astashova I. V. (ed.) Qualitative Properties of Solutions to Differential Equations and Related Topics of Spectral Analysis: scientific edition, M.: UNITY-DANA, 2012, pp. 22-290. (Russian),
- [3] I. Astashova, *On Existence of Non-oscillatory Solutions to Quasi-linear Differential Equations* Georgian Mathematical Journal **14**, 2 (2007), 223-238.

# ON THE SOLUTIONS TO GROWTH EQUATIONS AND SURFACE PATTERN MODELS

Gabriella Bognár, *Miskolc, Hungary*

The formation and spatio-temporal evolution of surfaces generated by deposition processes has recently attracted considerable interest. The dynamics of the surface morphology, e.g. in amorphous thin-film growth is dominated by the interplay of roughening, smoothing, and pattern forming processes. Molecular Beam Epitaxy (MBE) is often used to grow nanostructure on crystal surfaces. One of the crucial aspects of the evolution of the surface morphology during MBE growth process is its possible unstable character, due to deterministic mechanisms, which prevent the growing surface to stay parallel to the substrate. This phenomenon has turned out to be a source of a wide class of nonlinear dynamics, which varies from spatio-temporal chaos to the formation of stable structures [1].

One of the many challenges involved in applied mathematics and non-equilibrium physics is to predict the behavior of surface evolution, from the knowledge initial arbitrary profile and the scaling relationships between surface features in various growth regimes.

The theoretical description of the time evolution of an initially flat surface with the height function  $u(\mathbf{x}, t)$  of the growing surface at the position  $\mathbf{x} = (x, y)$  and time  $t$  is based on nonlinear partial differential equations. The coarsening processes and the diverging amplitude structures of amorphous thin film growth will be investigated through the surface growth model

$$\partial_t u = -\Delta [\nu u + \kappa \Delta u + \beta |\nabla u|^2] + \lambda |\nabla u|^2$$

with coefficients  $\nu, \kappa, \beta$  and  $\lambda$  introduced by Raible et al. [2] in one and two dimensions.

- [1] O. Pierre-Louis, C. Misbah, Y. Saito, J. Krug, P. Politi, *New Nonlinear Evolution Equation for Steps during Molecular Beam Epitaxy on Vicinal Surfaces*. Phys. Rev. Lett. **80** (1998), 4221–4224.
- [2] M. Raible, S.J. Linz, P. Hänggi, *Amorphous thin film growth: Minimal deposition equation*. Phys. Rev. E **62** (2000), 1691–1705.

# PERIODICITY IN THE BEVERTON–HOLT POPULATION MODEL

Martin Bohner, *Rolla, MO, USA*

We consider the Beverton–Holt population model and let both the carrying capacity and the inherent growth rate vary periodically. Versions of two so-called Cushing–Henson conjectures are presented. Dynamic analogues of the Beverton–Holt equation are considered, on arbitrary periodic time scales and also on the quantum time scale.

- [1] M.Bohner, H. Warth, *The Beverton–Holt dynamic equation* *Applicable Anal.* **86**(8) (2007), 1007–1015.
- [2] M.Bohner, R. Chiochan, *The Beverton–Holt quantum difference equation* *J. Biol. Dyn.* **7**(1) (2013), 86–95.
- [3] M.Bohner, S. Streipert, *The Beverton–Holt equation with periodic growth rate* *Int. J. Math. Comput.* **26**(4) (2015), 1–10.

## OSCILLATION AND NONOSCILLATION RESULTS FOR HALF-LINEAR EQUATIONS WITH DEVIATED ARGUMENT

Drábek Pavel, *Pilsen, Czech Republic*

This is a joint work with Alois Kufner and Komil Kuliev. As a birthday present for *Ondřej Došlý*, on the occasion of his  *jubilee*, we introduce oscillatory and nonoscillatory criteria for half-linear equations with deviated argument. Our method relies on the *weighted Hardy inequality*.

Let us consider the *half-linear equation with deviated argument*

$$(r(t)|u'(t)|^{p-2}u'(t))' + c(t)|u(\tau(t))|^{p-2}u(\tau(t)) = 0, \quad t \in (0, \infty), \quad (1)$$

where  $p > 1$ ,  $c: [0, \infty) \rightarrow (0, \infty)$  is continuous,  $c \in L^1(0, \infty)$ ,  $r: [0, \infty) \rightarrow (0, \infty)$  is continuously differentiable,  $\tau: [0, \infty) \rightarrow \mathbb{R}$  is continuously differentiable and increasing function satisfying  $\lim_{t \rightarrow \infty} \tau(t) = \infty$ .

Assume that (1) has at least one nonzero global solution defined on the entire interval  $(0, \infty)$ . We say that equation (1) is *nonoscillatory* (at  $\infty$ ) if for every global solution  $u = u(t)$  of (1) there exists  $T > 0$  such that  $u(t) \neq 0$  for all  $t > T$ . Otherwise, the equation (1) is called *oscillatory* (at  $\infty$ ), that is, there exists a global solution  $u = u(t)$  of (1) and a sequence  $(t_n)$  such that  $\lim_{n \rightarrow \infty} t_n = \infty$  and  $u(t_n) = 0$  for all  $n \in \mathbb{N}$ . We let  $p' = \frac{p}{p-1}$ .

**Theorem 1** (nonoscillatory criterion). *Let*

$$\limsup_{t \rightarrow \infty} \left( \int_0^t r^{1-p'}(s) \, ds \right) \left( \int_t^\infty c(s) \, ds \right)^{\frac{1}{p-1}} < \frac{(p-1)}{p^{p'}} \quad (2)$$

and

$$\limsup_{t \rightarrow \infty} \left( \int_0^{\tau(t)} r^{1-p'}(s) \, ds \right) \left( \int_t^\infty c(s) \, ds \right)^{\frac{1}{p-1}} < \frac{(p-1)}{p^{p'}}. \quad (3)$$

Then equation (1) is nonoscillatory.

**Theorem 2** (oscillatory criterion). *Let one of the following three cases occur:*

(i) *There exists  $T > 0$  such that for all  $t \geq T$  we have  $\tau(t) \geq t$  and*

$$\limsup_{t \rightarrow \infty} \left[ \left( \int_0^t r^{1-p'}(s) \, ds \right) \left( \int_t^\infty c(s) \, ds \right)^{\frac{1}{p-1}} + \left( \int_t^{\tau(t)} r^{1-p'}(s) \, ds \right) \left( \int_{\tau(t)}^\infty c(s) \, ds \right)^{\frac{1}{p-1}} \right] > 1.$$

(ii) *There exists  $T > 0$  such that for all  $t \geq T$  we have  $\tau(t) \leq t$  and*

$$\limsup_{t \rightarrow \infty} \left( \int_0^{\tau(t)} r^{1-p'}(s) \, ds \right) \left( \int_t^\infty c(s) \, ds \right)^{\frac{1}{p-1}} > 1.$$

(iii) *For any  $T > 0$  the function  $\tau(t) - t$  changes sign in  $(T, \infty)$  and either*

$$\liminf_{\substack{t \rightarrow \infty \\ t > \tau(t)}} \left( \int_0^{\tau(t)} r^{1-p'}(s) \, ds \right) \left( \int_t^\infty c(s) \, ds \right)^{\frac{1}{p-1}} > 1$$

or

$$\liminf_{\substack{t \rightarrow \infty \\ t < \tau(t)}} \left[ \left( \int_0^t r^{1-p'}(s) \, ds \right) \left( \int_t^\infty c(s) \, ds \right)^{\frac{1}{p-1}} + \left( \int_t^{\tau(t)} r^{1-p'}(s) \, ds \right) \left( \int_{\tau(t)}^\infty c(s) \, ds \right)^{\frac{1}{p-1}} \right] > 1.$$

Then equation (1) is oscillatory.

A typical example of  $\tau = \tau(t)$  is a linear function

$$\tau(t) = t - \tau, \quad \tau \geq 0 \quad \text{is fixed.}$$

Then (1) is half-linear equation with the *delay* given by fixed parameter  $\tau \geq 0$ . For this, rather special case, (2) implies (3), and only the case (ii) of Theorem 2 occurs. Hence we have the following corollary concerning the equation

$$(r(t)|u'(t)|^{p-2}u'(t))' + c(t)|u(t-\tau)|^{p-2}u(t-\tau) = 0, \quad t \in (0, \infty). \quad (4)$$

**Corollary 3** (equation with delay). *Let (2) hold. Then equation (4) with the delay  $\tau \geq 0$  is nonoscillatory. On the other hand, let*

$$\limsup_{t \rightarrow \infty} \left( \int_0^{t-\tau} r^{1-p'}(s) ds \right) \left( \int_t^\infty c(s) ds \right)^{\frac{1}{p-1}} > 1.$$

*Then equation (4) with the delay  $\tau \geq 0$  is oscillatory.*

- [1] O. Došlý and P. Řehák, Half-linear Differential Equations, North Holland Mathematic Studies 202, Elsevier, Amsterdam, Boston, New York, Tokyo, 2005.
- [2] B. Opic and A. Kufner, Hardy-Type Inequalities, Pitman Research Notes in Mathematics Series 279, Longman Scientific and Technical, Harlow, 1990.

## RELATIVE OSCILLATION OF LINEAR HAMILTONIAN DIFFERENTIAL SYSTEMS WITHOUT CONTROLLABILITY

**Julia Elyseeva, Moscow, Russia**

We develop the concept of relative oscillation for the pair of linear Hamiltonian differential systems

$$y' = J\mathcal{H}(t)y, \mathcal{H}(t) = \begin{bmatrix} -C(t) & A^T(t) \\ A(t) & B(t) \end{bmatrix}, \mathcal{H}(t) = \mathcal{H}^T(t), J = \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix} \quad (1)$$

and

$$\hat{y}' = J\hat{\mathcal{H}}(t)\hat{y}, \hat{\mathcal{H}}(t) = \begin{bmatrix} -\hat{C}(t) & \hat{A}^T(t) \\ \hat{A}(t) & \hat{B}(t) \end{bmatrix}, \hat{\mathcal{H}}(t) = \hat{\mathcal{H}}^T(t), t \in [a, \infty] \quad (2)$$

with the piecewise continuous blocks  $A(t), B(t), C(t), \hat{A}(t), \hat{B}(t), \hat{C}(t) : [a, \infty] \rightarrow \mathbb{R}^{n \times n}$  under the conditions

$$\mathcal{H}(t) - \hat{\mathcal{H}}(t) \geq 0, \quad \hat{B}(t) \geq 0, \quad t \in [a, \infty], \quad (3)$$

where  $A \geq 0$  means that the symmetric matrix  $A$  is nonnegative definite, and  $I, 0$  denotes the identity and zero matrices of appropriate dimensions. Recall that according to [1] system (1) is *relatively oscillatory* with respect to system (2) as  $t \rightarrow \infty$  if

$$\lim_{b \rightarrow \infty} (l(a, b, Y) - l(a, b, \hat{Y})) = \infty,$$

where  $l(a, b, Y), l(a, b, \hat{Y})$  are the multiplicities of focal points in  $(a, b]$  of conjoined bases  $Y(t), \hat{Y}(t)$  of systems (1), (2). Then, by Theorem 5.1 in [1] system (1) is *relatively oscillatory* with respect to system (2) if and only if some transformed Hamiltonian system associated with (1), (2) is oscillatory. The main result of the talk generalizes this result to the case of noncontrollable (see [2]) Hamiltonian systems (1)-(2). The consideration is based on the comparative index theory ([3], [4]) applied to the continuous case.

- [1] O. Došlý, *Relative Oscillation of Linear Hamiltonian Differential Systems* Math. Nachr., 2016, to appear
- [2] W. Kratz, *Definiteness of quadratic functionals* Analysis (Munich) **23**(2)(2003), 163–183.
- [3] Y.V. Eliseeva, *Comparison Theorems for Symplectic Systems of Difference Equations* Differ. Equ., **46**(9) (2010), 1339–1352.
- [4] J. Elyseeva, *A note on relative oscillation theory for symplectic difference systems with general boundary conditions* Appl. Math. Lett., **25** (2012), 1809–1814.



# MODIFIED RICCATI TECHNIQUE FOR HALF-LINEAR EQUATIONS: ORDINARY AND DELAY CASE

Simona Fišnarová, *Brno, Czech Republic*

In the talk, we recall the modified Riccati technique for the half-linear differential equation

$$(r(t)\Phi(x'))' + c(t)\Phi(x) = 0, \quad \Phi(x) = |x|^{p-2}x, \quad p > 1. \quad (1)$$

The method can be seen as a generalization of the well-known transformation technique from the linear case  $p = 2$ . It is based on transformation of the Riccati-type equation related to (1) and enables us to compare oscillatory properties of two equations of type (1) with different coefficients and different powers in the nonlinearity.

We present some applications of the method in the oscillation theory of (1) and discuss the extension of the method to the equation with delay

$$(r(t)\Phi(x'(\tau(t))))' + c(t)\Phi(x(\tau(t))) = 0, \quad \tau(t) \leq t.$$

## MULTIPLE ANTI-PERIODIC SOLUTIONS TO NONLINEAR FOURTH ORDER DIFFERENCE EQUATIONS

John R. Graef, *Chattanooga, TN, USA*

This is a joint work with Lingju Kong and Xueyan "Sherry" Liu. This paper is concerned with the existence of multiple anti-periodic solutions to a nonlinear fourth order difference equation. The analysis is based on variational methods and critical point theory. Clark's critical point theorem is used to prove the main results. An example illustrates the applicability of the results.

## $sl_2$ AND FRIENDS

Stefan Hilger, *Eichstaett, Germany*

After defining the Lie algebra  $sl_2(\mathbb{C})$  we will consider and study the so called  $(q, h)$ -deformed universal enveloping algebra. Then we will elucidate connections to scalar dynamic equations on the one hand and to ladders in quantum groups on the other hand.

This is partially a joint work with Dr. Galina Filipuk. The support of the Alexander-von-Humboldt Foundation is gratefully acknowledged.

# PICONE'S IDENTITY REVISITED

Jaroslav Jaroš, Bratislava, Slovakia

We will take a detour through the history and the latest generalizations and applications of famous Picone's identity which plays an important role in the comparison theory of linear and half-linear ordinary and partial differential equations of the second order.

In particular, we will show how the pseudo- $p$ -Laplacian version of this identity obtained by Prof. Ondřej Došlý in [3] (see also [1]) can be further generalized to unify (and extend) most of Picone-type formulas known up to the present date.

The main result asserts that if  $\xi \rightarrow A(x, \xi)$  is a continuously differentiable, convex and homogeneous function of degree  $p(x) > 1$  with  $p(x)$  continuously differentiable in  $\Omega \subset \mathbb{R}^n$  and  $u$  and  $v$  are differentiable in  $\Omega$  with  $v(x) \neq 0$  in  $\Omega$ , then

$$\begin{aligned} & A(x, \nabla u) - \frac{1}{p(x)} \left\langle \nabla_{\xi} A(x, \nabla v), \nabla \left( \frac{|u|^{p(x)}}{|v|^{p(x)-2v}} \right) \right\rangle \\ &= \Phi_A(u, v) - \frac{1}{p(x)} \frac{|u|^{p(x)}}{|v|^{p(x)-2v}} \ln \left( \frac{|u|}{|v|} \right) \langle \nabla_{\xi} A(x, \nabla v), \nabla p(x) \rangle, \end{aligned}$$

where  $\langle, \rangle$  denotes the inner product in  $\mathbb{R}^n$  and

$$\begin{aligned} \Phi_A(u, v) = A(x, \nabla u) + (p(x) - 1) A \left( x, \frac{u}{v} \nabla v \right) - \\ \left\langle \nabla_{\xi} A \left( x, \frac{u}{v} \nabla v \right), \nabla p(x) \right\rangle \geq 0. \end{aligned}$$

- [1] G. Bognár, O. Došlý *The application of Picone-type identity for some non-linear elliptic differential equations*, Acta Math. Univ. Comen., New Ser. **72** (2003), 45–57.
- [2] G. Bognár, O. Došlý, *Picone-type identity for pseudo  $p$ -Laplacian with variable power*, Electron. J. Differential Equations **2012** (2012), No. 174, 1–8.
- [3] O. Došlý, *The Picone identity for a class of partial differential equations*, Math. Bohem. **127** (2002), 581–589.
- [4] J. Jaroš, *Caccioppoli estimates through an anisotropic Picone identity*, Proc. Amer. Math. Soc. **143** (2015), 1137–1144.
- [5] J. Jaroš, *A-harmonic Picone's identity with applications*, Ann. Mat. Pura Appl. **194** (2015), 719–729.

# OSCILLATION AND SPECTRAL THEORY OF CONTINUOUS HAMILTONIAN AND DISCRETE SYMPLECTIC SYSTEMS WITH APPLICATIONS

Werner Kratz, *Ulm, Germany*

We consider continuous Hamiltonian differential and discrete symplectic eigenvalue problems with Dirichlet boundary conditions. We present the basic results on these eigenvalue problems in both cases, continuous and discrete, which are the *Oscillation Theorem*, *Rayleigh's Principle*, *Existence* of eigenvalues, the *Expansion Theorem* and *Completeness* of the eigenfunctions. The main tools for the proofs will be discussed, in particular: *Picone's Identity*, *l'Hospital's Rule* for matrices, and an *Index Theorem* for monotone matrixvalued functions. Concluding we present two applications.

- [1] M. Bohner, O. Došlý and W. Kratz, *Sturmian and spectral theory for discrete symplectic systems* TAMS 361 (2009), 3109–3123.
- [2] W. Kratz, *Quadratic Functionals in Variational Analysis and Control Theory*, Akademie Verlag 1995.

# DIFFERENTIAL EQUATIONS WITH QUEUEING DELAYS

Tibor Krisztin, *Szeged, Hungary*

We consider a system of delay differential equations modeling queueing processes. These type of processes appear in computer networks, road networks, or when customers are waiting for service, or patients for treatment.

The state-dependent time delays are determined by algebraic equations involving the length of the queues. For the length of the queues discontinuous differential equations hold.

An appropriate framework is formulated to study the problem, and it is shown that the solutions define a continuous semiflow in the phase space. We study the stability of equilibria, and prove the existence of slowly oscillating periodic solutions.

# ON ZEROS OF SOLUTIONS OF A CERTAIN NONLINEAR ORDINARY DIFFERENTIAL EQUATION

Alexander Lomtatidze, Brno, Czech Republic

In the present lecture we consider the nonlinear equation

$$u'' = p(t)|u|^\alpha |u'|^{1-\alpha} \operatorname{sgn} u, \quad (1)$$

where  $\alpha \in ]0, 1[$  and the function  $p \in ]a, b[ \rightarrow \mathbb{R}$  is locally integrable and satisfies

$$\int_a^b (s-a)^\alpha (b-s)^\alpha |p(s)| ds < +\infty.$$

We will present several sufficient conditions for the existence of a non-trivial solution of (1) with at least two zeros on  $[a, b]$ . In this case, equation (1) is referred as *conjugate on  $[a, b]$* . Problem of non-conjugacy of (1) will be discussed, as well.

- [1] T. Chantladze, N. Kandelaki, A. Lomtatidze, *On zeros of solutions of a second order singular half-linear equation*, Mem. Differential Equations Math. Phys. **17** (1999), 127–154.
- [2] O. Došlý, A. Lomtatidze, *Disconjugacy and disfocality criteria for second order singular half-linear differential equations*, Ann. Polon. Math. **72** (1999), No. 3, 273–284.

## ODE METHODS OUTSIDE THE ODE WORLD

Robert Mařík, Brno, Czech Republic

The half-linear ordinary differential equation

$$(r(t)\Phi(u))' + c(t)\Phi(u) = 0, \quad \Phi(u) = |u|^{p-2}u, \quad p > 1 \quad (1)$$

is the equation which attracted attention of many scientists not only in Brno, but also in many highly respected research groups or individuals over the world. The main research directions and methods of study have been established in the book [1], but the theory is still growing fast.

One of the neat properties of the ordinary half-linear differential equation (1) is the ambivalent character of this equation: some of the classical methods used in theory of differential equations allow smooth extension to the half-linear equation and some methods either fail or require a new

approach. This natural phenomenon can be observed also for further generalizations of (1). In the talk we will discuss some of the generalizations which tear (1) out from the world of ordinary differential equations and present the results achieved in this direction. The main interest will be mainly (but not only) in the partial differential equation

$$\operatorname{div} (A(x) \|\nabla u\|^{p-2} \nabla u) + c(x) \Phi(u) = 0$$

and the distribution of zeros near infinity (oscillatory properties).

- [1] O. Došlý, P. Řehák, *Half-linear Differential Equations*, North-Holland Mathematics Studies 202, Elsevier, 2005.

## FIRST ORDER DIFFERENCE SYSTEMS WITH MULTIPOINT BOUNDARY CONDITIONS

*Jean Mawhin, Louvain-la-Neuve, Belgium*

Using Brouwer degree and convex analysis, we obtain geometric conditions for the existence of solutions of multipoint boundary value problems for  $n$ -dimensional difference systems of the form

$$\begin{aligned} \Delta u(k) &= f(k, u(k)) \quad (k = 0, 1, \dots, T-1), \\ u(T) &= \sum_{k=0}^{T-1} g(k)u(k), \end{aligned}$$

when  $T$  is a positive integer,  $u : \{0, 1, \dots, T\} \rightarrow \mathbb{R}^n$ ,  $k \mapsto u(k)$ , and

$$\Delta u(k) := u(k+1) - u(k) \quad (k = 0, 1, \dots, T-1)$$

is the usual forward difference operator.

It is assumed that the continuous vector fields  $f(k, u)$  point outside a bounded convex set of  $\mathbb{R}^n$  on its boundary, and the continuous diagonal matrix function  $g(k)$  satisfies suitable conditions, which cover the periodic boundary conditions.

- [1] J. Mawhin, *First order difference systems with multipoint boundary conditions* J. Difference Equations Applic. (2016), to appear.

# EXPONENTIAL STABILITY OF SOLUTIONS OF SOME MULTI-FRACTIONAL DIFFERENTIAL EQUATIONS

Milan Medveď, Bratislava, Slovakia

In this talk we present a result on the exponential stability of the zero solution of the integro-differential equation:

$$\dot{x}(t) = Ax(t) + f(t, x(t), {}^{CF}I^{\alpha_1}x(t), \dots, {}^{CF}I^{\alpha_m}x(t)), \quad t > 0, x \in \mathbb{R}^n, \quad (1)$$

where

$${}^{CF}I^{\alpha_i}x(t) := \frac{M(\alpha_i)}{1 - \alpha_i} \int_0^t \exp\left[-\frac{\alpha_i}{1 - \alpha_i}(t - s)\right] x(s) ds,$$

$0 < \alpha_i < 1$ ,  $M(\alpha_i)$ ,  $i = 1, 2, \dots, m$  is a normalization function such that  $M(0) = M(1) = 1$ . This result is proved in the paper [2], accepted for publication. This is a joint work with Eva Brestovanská. An example of such type of equation can be obtained from the following fractionally perturbed pendulum equation

$$u''(t) = -\eta u'(t) - \omega^2 u(t) + g(t, x(t), {}^{CF}D^{\alpha_1}x(t), \dots, {}^{CF}D^{\alpha_m}x(t)), \quad (2)$$

where

$${}^{CF}D^{\alpha_i}x(t) := \frac{M(\alpha_i)}{1 - \alpha_i} \int_0^t \exp\left[-\frac{\alpha_i}{1 - \alpha_i}(t - s)\right] \dot{u}(s) ds$$

is the Caputo-Fabrizio fractional derivative of the function  $u$ , defined by M. Caputo and M. Fabrizio in the paper [1], writing it as a system with  $x(t) = (u(t), \dot{u}(t))$ . We also recall a result, proved in [3], on the exponential stability of the zero solution of an equation of the form (1), however with the Riemann-Liouville fractional integrals  $I^{\alpha_i}[g_i](t) := \frac{1}{\Gamma(\alpha_i)} \int_0^t (t - s)^{\alpha_i - 1} g_i(s, x(s)) ds$ ,  $0 < \alpha_i < 1$  instead of  ${}^{CF}I^{\alpha_i}x(t)$ , where  $g_i(t, x)$  is a nonlinear continuous mapping.

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- [2] E. Brestovanská, M. Medveď, *Exponential stability of solutions of a second order system of integrodifferential equations with the Caputo-Fabrizio fractional derivatives* Progr. Frac. Differ. Appl., **2** (3) (2016), 1-6.
- [3] M. Medveď, *Exponential stability of solutions of nonlinear differential equations with Riemann-Liouville fractional integrals in the nonlinearities* Proceedings of 4<sup>th</sup> Scientific Colloquium, Prague June 24-26 (2014), 10-20.

## THIRD-ORDER COUPLE SYSTEMS WITH DEPENDENCE ON THE FIRST DERIVATIVE

Feliz Minhós, Évora, Portugal

In this talk we present sufficient conditions for the solvability of the third order three point boundary value problem

$$\left\{ \begin{array}{l} -u'''(t) = f(t, v(t), v'(t)) \\ -v'''(t) = h(t, u(t), u'(t)) \\ u(0) = u'(0) = 0, u'(1) = \alpha u'(\eta) \\ v(0) = v'(0) = 0, v'(1) = \alpha v'(\eta), \end{array} \right.$$

The non-negative continuous functions  $f$  and  $h$  verify adequate superlinear and sublinear conditions,  $0 < \eta < 1$  and the parameter  $\alpha$  such that  $1 < \alpha < (1/\eta)$ . The arguments apply Green's function associated to the linear problem and the Guo–Krasnosel'skiĭ theorem of compression-expansion cones. The dependence on the first derivatives is overcome by the construction of an adequate cone and suitable conditions of superlinearity/sublinearity near 0 and  $+\infty$ .

## BOUNDEDNESS OF POSITIVE SOLUTIONS OF A SYSTEM OF NONLINEAR DELAY DIFFERENTIAL EQUATIONS

Nahed A. Mohamady, Veszprém, Hungary

This is a joint work with István Győri and Ferenc Hartung.

In our talk we give sufficient conditions for the the uniform permanence of the positive solutions of a system of nonlinear delay differential equations

$$\begin{aligned} \dot{x}_i(t) &= \sum_{j=1}^n \alpha_{ij}(t) h_{ij}(x_j(t - \tau_{ij})) - \beta_i(t) f_i(x_i(t)) + \rho_i(t), \\ &t \geq 0, \quad 1 \leq i \leq n \end{aligned} \tag{1}$$

with the initial condition

$$x_i(t) = \varphi_i(t), \quad -\tau \leq t \leq 0, \quad 1 \leq i \leq n, \tag{2}$$

where  $\tau = \max_{1 \leq i, j \leq n} \tau_{ij} > 0$  is a positive constant,  $\alpha_{ij}$ ,  $h_{ij}$ ,  $\beta_i$ ,  $f_i$ ,  $\rho_i \in C(\mathbb{R}_+, \mathbb{R}_+)$ ,  $1 \leq i, j \leq n$ ,  $\mathbb{R}_+ := [0, \infty)$  and  $\varphi_i \in C([-\tau, 0], \mathbb{R}_+)$

with  $\varphi_i(0) > 0$ ,  $1 \leq i \leq n$ . As our main result we prove, under certain conditions, that the positive solutions  $x(\varphi) = (x_1(\varphi), \dots, x_n(\varphi))$  of the initial value problem (1) and (2) satisfy

$$\underline{x}_i^* \leq \liminf_{t \rightarrow \infty} x_i(\varphi)(t) \leq \limsup_{t \rightarrow \infty} x_i(\varphi)(t) \leq \bar{x}_i^*, \quad 1 \leq i \leq n, \quad (3)$$

where  $(\underline{x}_1^*, \dots, \underline{x}_n^*)$  and  $(\bar{x}_1^*, \dots, \bar{x}_n^*)$  are unique positive solutions of a certain associated nonlinear algebraic system. Several examples are given to demonstrate the main result.

## EXTREMAL SOLUTIONS OF MEASURE DIFFERENTIAL EQUATIONS AND APPLICATION TO IMPULSIVE SYSTEMS

*Giselle A. Monteiro, Košice, Slovakia*

This is a joint work with Antonín Slavík (Charles University in Prague, Czech Republic). We investigate the existence of greatest and least solutions for the so-called measure differential equations, i.e., integral equations of the form

$$y(t) = y_0 + \int_a^t f(y(s), s) dg(s), \quad (1)$$

where the integral on the right-hand side is the Kurzweil-Stieltjes integral. Recalling that equations with impulses represent a special case of (1), we derive new results about extremal solutions of impulsive systems.

*Acknowledgement:* Financed by the SASPRO Programme, co-financed by the European Union and the Slovak Academy of Sciences.

## DISCONJUGACY VIA LYAPUNOV AND VALLÉE-POUSSIN TYPE INEQUALITIES FOR FORCED DIFFERENTIAL EQUATIONS

*Abdullah Özbekler, Ankara, Turkey*

This is a joint work with Prof. Ravi P. Agarwal. In this talk, in the case of oscillatory potentials, we present some new Lyapunov and Vallée-Poussin type inequalities for second order forced differential equations. No sign restriction is imposed on the forcing term. The obtained inequalities generalize and complement the existing results in the literature.



# LINEARIZED OSCILLATION FOR NONAUTONOMOUS DELAY DIFFERENTIAL EQUATIONS

Mihály Pituk, *Veszprém, Hungary*

We present a linearized oscillation criterion for a nonautonomous scalar delay differential equation. The hypotheses include a weak recurrence property of the coefficient of the linearized equation. The importance of the recurrence assumption will be shown by an example.

## ON SOME MODELS OCCURRING IN SYNCHRONIZATION

Vladimir Rasvan, *Craiova, Romania*

The starting point as well as the motivation of this paper is given by the old problem of Huygens on synchronization of two oscillators (e.g. mechanical pendula) connected through a distributed environment. We present here two mechanical models and an electrical one.

The first of the mechanical models as well as the electrical one consider two oscillators (pendula or electronic oscillators respectively) coupled to an elastic string or to a lossless LC line respectively, both of infinite length. The second mechanical model is an oscillatory mechanical structure with two oscillators coupled to a finite elastic rod.

The mathematical development strongly relies on the association of some functional differential equations of neutral type having the structure

$$\begin{aligned} \dot{x} &= A_0x(t) + A_1x(t - \tau) - \sum_1^m b_{1k}\phi_k(c_k^*x(t)) + f(t) \\ y(t) &= A_2x(t) + A_3x(t - \tau) - \sum_1^m b_{2k}\phi_k(c_k^*x(t)) + g(t) \end{aligned} \tag{1}$$

where  $f(t)$ ,  $g(t)$  can be (almost) periodic and  $\phi_k(\cdot)$  are sector restricted. Only the electrical model has  $A_3$  - a Schur matrix thus allowing standard forced oscillations i.e. synchronization. The mechanical models have  $A_3$  with eigenvalues on the unit circle (critical case) what implies some (pseudo-) "complex behavior".

# STABILITY IN DEMAND-INVENTORY MODEL

Ewa Schmeidel, *Białystok, Poland*

This is a joint work with Piotr Hachuła and Magdalena Nockowska-Rosiak. Ma and Feng in [3] proposed the dynamical model of demand and inventory with mechanism of demand stimulation and inventory limitation. The model describes demand and inventory of a product at one echelon of supply chain - at retailer, and it takes a form of the following system of difference equations

$$\begin{cases} D_{n+1} &= \left[ \frac{AT}{(A+1)T - S_n} \right]^k D_n \\ S_{n+1} &= S_n - D_n + \check{D}_n \\ \check{D}_{n+1} &= \alpha D_n + (1 - \alpha)\check{D}_n \end{cases} \quad (1)$$

where:  $n \in \mathbb{N}$ ,  $S_n$  is a stock volume,  $D_n \geq 0$  is a demand volume,  $\check{D}_n \geq 0$  is a forecast of demand at  $n$  and ordered placed by a retailer at a manufacturer, moreover by assumption of unlimited capacity it is also delivered quantity at  $n$ ,  $A > 0$  is a parameter for discount steering,  $T > 0$  is a parameter for defining the target stock,  $k > 0$  is price elasticity coefficient that regulates dependence between price, discount and demand,  $\alpha \in (0, 1)$  is a forecast smoothing coefficient. Stability of equilibrium points of this model will be presented.

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- [2] P. Hachuła, M. Nockowska-Rosiak and E. Schmeidel, *An analysis of dynamics of discrete demand-inventory model with bifurcation diagrams and phase portraits* Book Series: AIP Conference Proceedings (accepted).
- [3] J. Ma and Y. Feng, *The study of the chaotic behavior in retailer's demand model* Discrete Dyn. Nat. Soc. Article ID **792031** (2008).
- [4] M. Nockowska-Rosiak, P. Hachuła and E. Schmeidel, *Stability of equilibrium points of demand-inventory model in a specific business case* (submitted).

# WELL-POSEDNESS AND MAXIMUM PRINCIPLES FOR LATTICE REACTION-DIFFUSION EQUATIONS

Petr Stehlík, *Pilsen, Czech Republic*

This is a joint work with Antonín Slavík and Jonáš Volek. We consider nonautonomous reaction-diffusion on lattices and graphs. We prove the local existence and global uniqueness of bounded solutions, as well as continuous dependence of solutions on the underlying time structure and on initial conditions. Next, we obtain the weak maximum principle, which enables us to get global existence of solutions. Finally, we provide the strong maximum principle, which exhibits an interesting dependence on the time structure. Our results are illustrated by the autonomous Fisher and Nagumo lattice equations.

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- [2] A. Slavík, P. Stehlík, *Explicit solutions to dynamic diffusion-type equations and their time integrals*. Appl. Math. Comput. **234** (2014), 486–505.
- [3] P. Stehlík, J. Volek, *Transport equation on semidiscrete domains and Poisson-Bernoulli processes*, J. Difference Equ. Appl. **19** (2013), no. 3, 439–456.

# LINEARIZATION TECHNIQUES OF HALF-LINEAR DIFFERENTIAL EQUATIONS

Mehmet Ünal, *Sinop, Turkey*

This is a joint work with Prof Ondřej Došlý. Oscillatory properties of the half-linear second-order differential equation

$$(r(t)\Phi(x'))' + c(t)\Phi(x) = 0, \Phi(x) = |x|^{p-2}x, \quad p > 1 \quad (1)$$

are investigated, where (1) is viewed as a perturbation of an equation of the same form, and these properties are compared with the oscillatory behavior of a certain associated linear second-order differential equation.

- [1] O. Došlý, *Perturbations of the half-linear Euler–Weber type differential equation*, J. Math. Anal. Appl. **323** (2006) 426–440.
- [2] O. Došlý, *A remark on the linearization technique in half-linear oscillation theory*, Opuscula Math. **26** (2006) 305–315.
- [3] F. Gesztesy and M. Ünal, *Perturbative Oscillation Criteria and Hardy Type Inequalities*, Math. Nachr. **189** (1998), 121–144.

# LANDESMAN-LAZER CONDITIONS FOR DIFFERENCE EQUATIONS INVOLVING SUBLINEAR PERTURBATIONS

Jonáš Volek, *Pilsen, Czech Republic*

We study the existence and uniqueness for discrete Neumann and periodic problems. We consider both ordinary and partial difference equations involving sublinear perturbations. All the proofs are based on reformulating these discrete problems as a general singular algebraic system. Firstly, we use variational techniques (specifically, the Saddle Point Theorem) and prove the existence result based on a type of Landesman-Lazer condition. Then we show that for a certain class of bounded nonlinearities this condition is even necessary and therefore, we specify also the cases in which there does not exist any solution. Finally, we study the uniqueness.

## OSCILLATION CRITERIA FOR SECOND-ORDER NONLINEAR DIFFERENTIAL EQUATIONS WITH $p$ -LAPLACIAN

Naoto Yamaoka, *Sakai, Japan*

This talk is based on [1]. We are concerned with the oscillation problem for the second-order nonlinear differential equation

$$(t^{\alpha-1}\Phi_p(x'))' + t^{\alpha-1-p}f(x) = 0, \quad (1)$$

where  $\alpha \in \mathbb{R}$ ,  $p > 1$ ,  $\Phi_p(x) = |x|^{p-2}x$  and  $f(x)$  is continuous on  $\mathbb{R}$  and satisfies the signum condition  $xf(x) > 0$  if  $x \neq 0$ , but is not assumed to be monotone. The term  $(\Phi_p(x'))'$  is called the one-dimensional  $p$ -Laplacian operator, and it is known that equation (1) has a close relationship to elliptic equations with  $p$ -Laplacian. The relation between the constants  $\alpha$  and  $p$  influences the asymptotic behavior of solutions of equation (1) as  $t \rightarrow \infty$ . By means of this fact, Riccati technique, and phase plane analysis of a system, (non)oscillation criteria for equation (1) are established.

- [1] O. Došlý and N. Yamaoka, *Oscillation constants for second-order ordinary differential equations related to elliptic equations with  $p$ -Laplacian*, *Nonlinear Anal.* **113** (2015), 115–136.

## EXISTENCE RESULTS FOR THE BOUNDARY VALUE PROBLEM RELATED TO THE LIEBAU PHENOMENON

Mirosława Zima, *Rzeszów, Poland*

The talk is based on the joint paper with José Ángel Cid, Gennaro Infante and Milan Tvrđý [1]. We will discuss the existence and localization of positive solutions for  $x''(t) + ax'(t) = r(t)x^\alpha - s(t)x^\beta$  subject to  $x(0) = x(T)$ ,  $x'(0) = x'(T)$ . An interesting feature here is that the problem is related to the Liebau phenomenon. The main tool is the Krasnoselskii fixed point theorem on cone expansion and compression of the norm type.

- [1] J. A. Cid, G. Infante, M. Tvrđý and M. Zima, A topological approach to periodic oscillations related to the Liebau phenomenon, *J. Math. Anal. Appl.* **423** (2015) 1546–1556.



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