Semilinear elliptic problems involving Leray-Hardy and measure data

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We discuss isolated singular solution of nonhomogeneous elliptic problem with the operator $u \mapsto \mathcal{L}_\mu u = -\Delta u + \frac{\mu}{|x|^2} u$, and then we show the existence and uniqueness of weak solutions for

$$\mathcal{L}_\mu u + g(u) = \nu \quad \text{in} \quad \Omega, \quad u = 0 \quad \text{on} \quad \partial\Omega,$$

where $\Omega \subset \mathbb{R}^N$ is a bounded smooth domain such that $0 \in \Omega$, $\mu \geq -\frac{(N-2)^2}{4}$ is a constant, g a continuous nondecreasing function satisfying some integral growth condition, ν is a Radon measure in Ω.

We show that the situation differs according ν is diffuse or concentrated at the origin. When g is a power we introduce a capacity framework to find necessary and sufficient condition for solvability.