MARIE-FRANÇOISE DAZA, LAURENT ARIZA: EL AMOR EN LOS TIEMPOS DEL CORONAVIRUS

LUCIO BOCCARDO

Two maximum principle for two friends

In an elliptic boundary value problem like

$$\begin{cases} A(u) = f(x), & \text{in } \Omega, \\ u = 0, & \text{on } \partial\Omega, \end{cases} \tag{0.1}$$

where Ω is a bounded open set in $\mathbb{R}^N$, A is a second order elliptic operator in divergence form and $f(x) \geq 0$ (of course not zero a.e.), the Weak Maximum Principle states that $u(x) \geq 0$, and it is zero at most in a zero measure set; whereas the Strong Maximum Principle states that the set where $u(x) = 0$ is even "smaller than a zero measure set" (e.g. empty).

The presence of a lower order term can destroy the Maximum Principle property (see [6] and the introduction of [5]).

Nevertheless, we will prove a Maximum Principle for two boundary value problems having a lower order term.

1. A weak maximum principle for Marie-Françoise

A weak maximum principle is proved for the Dirichlet problems

$$u \in W_0^{1,2}(\Omega): \ -\mathrm{div}(M(x)\nabla u) + u = -\mathrm{div}(u\,E(x)) + f(x)$$

$$\psi \in W_0^{1,2}(\Omega): \ -\mathrm{div}(M(x)\nabla \psi) + \psi = E(x)\cdot\nabla\psi + g(x)$$

under the assumptions (on E, $f\,g$) of the existence theorems.

2. A strong maximum principle for Laurent

A strong maximum principle for some quasilinear elliptic equations with lower order terms having natural (i.e. quadratic) growth with respect to the gradient is proved. One of the main motivations for the interest in quasilinear elliptic equations with lower order terms having quadratic growth with respect to the gradient comes from the Calculus of Variations.

An important case is

$$\begin{cases} -\mathrm{div}([a(x)+|u|^r]\nabla u) + \dfrac{r}{2}u|u|^{r-2}|\nabla u|^2 = f & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega. \end{cases}$$

where $0 < \alpha \leq a(x) \leq \beta$, and $r > 1$; the right hand side $f(x)$ can be very singular (regularizing effect since [4]).

References

[1] L. Boccardo. Weak maximum principle for Dirichlet problems with convection or drift terms.
[2] L. Boccardo, L. Orsina. Strong maximum principle for some quasilinear Dirichlet problems having natural growth terms.
[3] L. Boccardo. A contribution to the theory of quasilinear elliptic equations and application to the minimization of integral functionals. *Milan J. Math.*, **79** (2011), 193–206.
[4] L. Boccardo, T. Gallouët. Strongly nonlinear elliptic equations having natural growth terms and L^1 data. *Nonlinear Anal.*, **19** (1992), 573–579.
[5] P. Pucci, V. D. Rădulescu. The maximum principle with lack of monotonicity. *Electron. J. Qual. Theory Differ. Equ.*, (2018), Paper No. 58, 11.
[6] J. L. Vázquez. A strong maximum principle for some quasilinear elliptic equations. *Appl. Math. Optim.*, **12** (1984), 191–202.

L.B. Istituto Lombardo Accademia di Scienze e Lettere - Sapienza Università di Roma
E-mail address: boccardo@mat.uniroma1.it