

# Beyond the unique continuation: “flat solutions” for reactive slow diffusion and the confinement singular potentials for the Schrödinger equation

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**Abstract:** Solutions with compact support for some nonlinear elliptic and parabolic equations, and many other free boundary problems, are formulations for which the Unique Continuation Principle, in its several versions, fails. In such problems (which have attracted the attention of M.F. Bidaut-Veron, L. Veron, and many other specialists) the solution  $u$  and its normal derivative vanish on a region of the boundary (which leads to the definition of a “flat solution” of the corresponding equation).

In this talk I will present, in a very sketched way, some recent results in this direction, trying to show how many open problems of this nature still remain as a source of current research. More specifically, I will report on some results concerning the following problems:

- I) Stable flat solutions of  $u_t - \Delta u^m + u^a = \lambda u^b$  for  $0 < a < b < m$  under the stability condition  $2(m+a)(m+b) - N(m-a)(m-b) < 0$  (joint work with J. Hernández and Y. Sh. Ilyasov),
- II) Flat solutions to  $\mathbf{i} \frac{\partial \psi}{\partial t} = -\Delta \psi + V(x)\psi$  in  $\mathbb{R}^N$ , for  $V(x) \geq Cd(x, \partial\Omega)^{-\alpha}$ , with  $\alpha \geq 2$ , for a given bounded domain  $\Omega$  (my own research, continued in collaboration with J. M. Rakotoson, D. Gómez-Castro, R. Temam and J. L. Vázquez).