

Eduard Čech Center for Algebra and Geometry

2006 report summary (the full report had to be submitted in Czech)

The research was continued in three main directions

- A. Differential Geometry including aspect of algebraical and topological character
- B. Algebraic and categorical structures including some applications in informatics
- C. Mathematical logic and proof complexity

The ECC project, in its second year, has provided a stabilized institutional and organizational background for the 6 PhD positions and the supporting senior staff at all three institutions (Masaryk University in Brno, Charles University in Prague, and Mathematical Institute of the Czech Academy of Sciences in Brno and Prague).

The main goal of the project is to foster international cooperation, in particular among the young post-doctoral members of the team, but also supporting wider development at the level of the doctoral and post-doctoral research at all three institutions. We may also witness further convergence of the interests between the teams in ECC. The workshop on operadical analysis and algebra as well as the second ECC general meeting in Telc were particularly good examples in 2006.

For a short and transparent account, I comment in more detail the achievements of the individual post-doctoral fellows:

Masaryk University in Brno

The position of **Mgr. Michal Bulant, PhD (team B)** was terminated by February 2006. He dealt with algebraic number theory together with R. Kučera (Brno) a C. Greither (Mnichov). His paper listed already in the 2005 report has not been published yet. *Michal Bulant is now at permanent position at the Masaryk University.*

Doojin Hong, PhD (team A) finished his position at ECC by July 31, 2006. He worked mainly on spectral properties of invariant differential operators on conformal geometries (in particular in cooperation with the recently passed Tom Branson of Iowa). In Brno, he succeeded in finding explicit formulae for intertwining operators on functions and differential forms on $S^p \times S^q$ in the metric signature (p, q) , and those on tensor-spinors for the Lorentzian metric on $S^l \times S^m$. He wrote two papers with the support of the ECC:

- (with Thomas Branson) *Spectrum generating on twistor bundle*, Proceedings Geometry and Physics, 2006, Rendiconti del Circ. Mat. Palermo, to appear, <http://arxiv.org/abs/math.DG/0606524> .
- (with Thomas Branson) *Translation to bundle operators*, Preprint math.DG/0606552, submitted for the Midwest Geometry Conference 2007, Iowa, <http://arxiv.org/abs/math.DG/0606552> .

Two his journeys to USA were supported by the ECC, the first one to University of Wisconsin at Fond du Lac, the second one to the IMA Summer program Symmetries and overdetermined systems of PDE in Minneapolis, Minnesota. *Doojin Hong is currently holding a research position at the University of Seoul, Korea.*

Immediately after his PhD in Auckland, New Zealand, **Mgr. Josef Šilhan, PhD (team A)** commenced position at the ECC by August 1, 2006. Very quickly he got an active member of

the team and he continued in his earlier cooperation with Rod Gover (Auckland), who also was one of the guests of the ECC during 2006. Josef completed two papers:

- (with Gover, R.); *The conformal Killing equation on forms - prolongations and applications*, Diff. Geom. Appl., to appear, <http://arxiv.org/abs/math.DG/0601751>
- (with Gover, R.) *A decomposition theorem for linear operators; applications to Einstein manifolds*, <http://arxiv.org/abs/math.AC/0701377>

while another one has been essentially finished:

- (with Gover, R.) *Detour complexes on conformally Einstein manifolds*, in progress and further research leading to generalizations of the remarkable constructions from his PhD Thesis for more general geometries has been started.

With the ECC support, Josef attended the Seminar "Representation Theory and Prehomogeneous Vector Spaces", (9.9-13.9, Institut de Recherche Mathématique Avancée in Strasbourg) and the Workshop "Geometry of vector distributions, differential equations, and variational problems" (13.12.-15.12., SISSA, Trieste), where he contributed an invited lecture "The conformal Killing equation on forms - prolongations and applications".

Valentina Rossi, PhD (team B), arrived to Brno shortly after defending her PhD on categorical Galois theory. Factorization systems played an important role there and she continued in this direction under the supervision of J. Rosický. The topics were largely broadened, which requested an extensive study of the literature. This has become a solid fundament for future cooperation with J. Rosický focused on possible extension of the "tilting" theory from modules to non-commutative situations. V. Rossi also presented two talks at the Algebraic Seminar in Brno (Admissible Galois structures and coverings in regular Mal'cev categories, 25.5. a Torsion theories and Galois coverings of topological groups, 15.6.). *Valentina stayed in Brno for 4 months.*

Similarly, **Tim van der Linden, PhD (team B)** arrived to Brno right after his PhD defense. His thesis dealt with extension of group homologies to semiabelian categories. Under leadership of J. Rosický he focused on the questions of general semiabelian theory, but based on rings. A fruitful cooperation with J. Rosický will very probably result in a joint paper during 2007. Tim also continued his previous work and finished one publication:

- (with T. Everaert, M. Gran), *Higher Hopf formulae for homology via Galois Theory*, preprint <http://arxiv.org/abs/math.AT/0701815>, to appear.

Tim had talks on the algebraic seminar in Brno (Homology in semi-abelian categories, 12.10., H vs. K: homotopy of rings, 7.12). *Tim stayed in Brno for 3 months.*

Among the applicants, also **Lukáš Vokřínek, PhD**, was offered a position. He is actually at Masaryk University, working together with the team A at ECC, but he is financed from other sources.

Charles University in Prague

Mgr. Pavel Příhoda, PhD (team B), spent one year at the ECC, leaving by September 15, 2006. Currently he is at another research position in Barcelona. At ECC, he finished two papers:

- *Projective modules are determined by radical factors*, accepted in Journal of Pure and Applied Algebra (see [dx.doi.org/10.1016/j.jpaa.2006.12.004](https://doi.org/10.1016/j.jpaa.2006.12.004))
- *Fair-sized projective modules*, submitted,

while another paper co-authored by L. Facchinim is in final stage of preparation.

Following the decision by the Board, **Marcin Kozik, PhD (team B)** commenced his one year position by October 1, 2006 (coming from the Jagelonian University in Cracow). Still in 2006 he finished the paper:

- *A finite set of functions with an EXPTIME composition problem*, accepted in SIAM Journal on Computing.

Due to excellent results, Marcin Kozik has been invited to plenary talks at conferences in 2007 – ALGORITHMIC COMPLEXITY AND UNIVERSAL ALGEBRA, in July in Szeged, Hungary, and WORKSHOP ON UNIVERSAL ALGEBRA AND THE CONSTRAINT SATISFACTION PROBLEM, in June in Nashvillu, USA.

Jan Trlifaj a R. Goebel (team B) finished their monograph

- *Approximations and Endomorphism Algebras of Modules*, 2006, Walter de Gruyter, Berlin, New York, ISBN 978-3-11-011079-1.

with partial support of ECC (stay of R. Goebel in Prague). The short stay of V. Repnitsky at ECC resulted in joint work with J. Tůma (team B)

- *Intervals in subgroup lattices of countable locally finite groups*, submitted in Algebra Universalis.

The ECC support had also be essential in 2005 for the paper

- *Subdirectly irreducible non-idempotent left distributive left quasigroups*, accepted in Communications in Algebra.

by **David Stanovský (team B)**.

Based on the decision of the Board, the post-doctoral position of **Alberto Damiano, PhD (team A)**, commencing by August 1, 2005 (after getting PhD at George Mason University, Fairfax - VA, USA), was prolonged by another year. He studied the Rarita-Schwinger operators and similar operators in more Clifford variables. He succesfully applies a blend of methods from algebraic analyzis, computer assisted computations (Cocoa) and representational theoretic techniques. During a visit by I. Sabadini, they proved that certain complexes built by methods of algebraic analysis and representation theory coincide, which leads to powerfull combinations of the methods. Two publications have been submitted:

- *Algebraic analysis of the Rarita-Schwinger system in real dimension 3*
- *Szygies of multi-variable higher spin Dirac operators on R^3*

The ECC support for allowing Alberto to atted some lecturing abroad: Milano (March 2006); the conference „Function theory in higher dimension“ in Tampere, (June 2006), Genua (July 2006).

The visit of R. Gover in Prague and Brno lead also to the paper:

- A.R. Gover, P. Somberg, V. Souček, *Yang-Mills detour complexes and conformal geometry*, preprint <http://arxiv.org/abs/math.DG/0606401>

Mathematical Institute, Czech Academy of Sciences

The one year stay at ECC of **Rafal Walczak, PhD. (team A)** ended by 31. 8. 2006. He worked together with J. Vanžura on differential 3-forms on 7-dimensional manifolds. The results are contained Rafal's the paper:

- (with J. Vanžura) *Orbit space of the space of 3-forms*, zasláno do Linear and Multilinear Algebra.

R. Walczak has currently a permanent position at the Univerzity of Opole.

The topic has been also followed by Hong Van Le with results in:

- *Construction of closed 3-forms of G2-type on $S^3 \times S^4$* submitted to Journal of Geometry and Physics.

Allan Skelley (team C), who worked in ECC until September 30, 2006, and **Neil Thapen (team C)**, commencing by September 1, 2006 had been working together before. Both cooperate with J. Krajíček. Several results of this cooperation were presented on a workshop in Cambridge in Spring 2006 and they are contained in the paper:

- *J. Krajíček; A. Skelley; N. Thapen: NP search problems in low fragments of bounded arithmetic*, vyjde v Journal of Symbolic Logic.

Further results are in

- *Skelley; N. Thapen: A resolution lower bound for a principle capturing the hardness of depths-1 LK*". Článek byl nabídnut do sborníku symposia Symposium on Logic in Computer Science 2007.

N. Thapen worked also with L. Kolodziejczyk, who visited ECC from Warsaw. Their results appear in the papers:

- *L.A. Kolodziejczyk; N. Thapen: The polynomial and linear hierarchies in models where the weak pigeonhole principle fails*, zaslaný do Journal of Symbolic Logic
- *L.A. Kolodziejczyk; N. Thapen: The polynomial and linear hierarchies in V^0* , přijato na konferenci Computability in Europe 2007 v Sieně ve formě zvané přednášky. Zároveň je článek přijatý do sborníku této konference.

A. Skelley finished also some research continuing his PhD and the results were presented at a conference in Wales and will appear in

- *Skelley: Third order computation and bounded arithmetic*, to appear in a special issue of Journal of Logic and Computation.