# Workshop on Neurogeometry

## Masaryk University, Jesuit College in Telč

## November 15 – 17, 2012

## 

This is preliminary programme. To be completed and partially confirmed.

**Title:  *TBA***

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**Abstract:**

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**Title:  *Curve cuspless reconstruction via sub-Riemannian geometry***

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**Abstract:** We consider the problem of minimizing $\int\_0^T (1 + K^2(s))^(1/2) ds$ for a planar curve having fixed initial and final positions and directions. The total length l is free. Here s is the variable of arclength parametrization and K(s) is the curvature of the curve. This problem comes from a model of geometry of vision due to Petitot, Citti and Sarti. Its main feature is that, if for a certain choice of boundary conditions there exists a minimizer, then this minimizer is smooth and has no cusp. However, not for all choices of boundary conditions there is a global minimizer. We study existence of local and global minimizers for this problem. We prove that if for a certain choice of boundary conditions there is no global minimizer, then there is neither a local minimizer nor a stationary curve (geodesic). We give properties of the set of boundary conditions for which there exists a solution of the problem. Finally, we present numerical computations of this set.

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[[[[**Title:  *Sub-Riemannian Geometric Control and PDE's on Lie Groups with Applications to Medical Imaging***](mailto:ugo.boscain@polytechnique.edu)](mailto:ugo.boscain@polytechnique.edu)](mailto:ugo.boscain@polytechnique.edu)](mailto:ugo.boscain@polytechnique.edu)

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[[[[**Title: *Riemann-Finsler Geometry and its Applications to Diffusion Magnetic Resonance Imaging***](mailto:R.Duits@tue.nl)](mailto:R.Duits@tue.nl)](mailto:ugo.boscain@polytechnique.edu)](mailto:ugo.boscain@polytechnique.edu)

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[[[[**Abstract:**](mailto:R.Duits@tue.nl)[Riemannian geometry has become a popular mathematical framework for the analysis of diffusion tensor images (DTI) in diffusion weighted magnetic resonance imaging (DWMRI). If one declines from the a priori constraint to model local anisotropic diffusion in terms of a 6-degrees-of-freedom rank-2 DTI tensor, then Riemann-Finsler geometry appears to be the natural extension. As such it provides an interesting alternative to the Riemannian rationale in the context of the various high angular resolution diffusion imaging (HARDI) schemes proposed in the literature. The main advantages of the proposed Riemann-Finsler paradigm are its manifest incorporation of the DTI model as a limiting case via a ``correspondence principle'' (operationalized in terms of a vanishing Cartan tensor), and its direct connection to the physics of DWMRI expressed by the (appropriately generalized) Stejskal-Tanner equation and Bloch-Torrey equations furnished with a diffusion term.](mailto:R.Duits@tue.nl)](mailto:R.Duits@tue.nl)](mailto:R.Duits@tue.nl)](mailto:R.Duits@tue.nl)

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[[[**Title:  Color space**](mailto:L.M.J.Florack@tue.nl)](mailto:R.Duits@tue.nl)](mailto:R.Duits@tue.nl)

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[[[**Abstract:** I will discuss the color vision and its relation to the 3-dimesional color space. I will review different coordinate systems and Riemannian metrics on this space.](mailto:L.M.J.Florack@tue.nl)](mailto:L.M.J.Florack@tue.nl)](mailto:L.M.J.Florack@tue.nl)

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[[[[**Title:**](mailto:L.M.J.Florack@tue.nl) ***An algorithm for image reconstruction based on hypoelliptic diffusion***](mailto:galaev@math.muni.cz)](mailto:L.M.J.Florack@tue.nl)](mailto:L.M.J.Florack@tue.nl)

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[[[[**Abstract:**](mailto:L.M.J.Florack@tue.nl)We study an algorithm for image reconstruction based on a model of geometry of vision due to Petitot, Citti, and Sarti. One of the main features of this model is that the primary visual cortex V1 lifts an image from R^2 to the bundle of directions of the plane. Neurons are grouped into orientation columns, each of them corresponding to a point of this bundle. In this model a corrupted image is reconstructed by minimizing the energy necessary for the activation of the orientation columns corresponding to regions in which the image is corrupted. The minimization process intrinsically defines a hypoelliptic heat equation on the bundle of directions of the plane. We present an algorithm which allows us to perform image reconstruction on real nonacademic images. A very interesting point is that this algorithm is massively parallelizable and needs no information on where the image is corrupted.](mailto:L.M.J.Florack@tue.nl)](mailto:L.M.J.Florack@tue.nl)](mailto:galaev@math.muni.cz)

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[[[**Title:  *Luneburg theory***](mailto:gauthier@univ-tln.fr)](mailto:L.M.J.Florack@tue.nl)](mailto:L.M.J.Florack@tue.nl) ***of binocular vision***

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[[[**Abstract:**](mailto:gauthier@univ-tln.fr)](mailto:gauthier@univ-tln.fr)](mailto:gauthier@univ-tln.fr)We consider the problem of defining a metric on the space of visual sensations. Upon defining such a metric we will use the result to define geodesics of this space, and apply this to optical illusions.

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[[[[**Title: *On conformal differential invariants in neurogeometry***](mailto:gauthier@univ-tln.fr)](mailto:cgustad@gmail.com)](mailto:cgustad@gmail.com)](mailto:gauthier@univ-tln.fr)

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[[[[**Abstract:**](mailto:gauthier@univ-tln.fr)[The role of conformal geometry and conformal differential invariants in neurogeometry based on Einstein-Maxwell equations will be discussed. The structure of the differential invariant algebra will be found and its completeness will be proved. Some conformally invariant differential equations and related geometrical structures will be shown.](mailto:gauthier@univ-tln.fr)](mailto:gauthier@univ-tln.fr)](mailto:gauthier@univ-tln.fr)](mailto:cgustad@gmail.com)

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[[[[**Title: *Overview on Geometries of Shape Spaces and Diffeomorphism groups***](mailto:gauthier@univ-tln.fr)](mailto:gauthier@univ-tln.fr)](mailto:gauthier@univ-tln.fr)](mailto:gauthier@univ-tln.fr)

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[[[[**Abstract:**](mailto:gauthier@univ-tln.fr)[For a compact manifold $M$ and a manifold $N$, we consider the spaces of embeddings and of immersions of $M$ into $N$, the regular Lie groups of diffeomorphisms of $M$ and of those diffeomorphisms on $N$ which fall rapidly to the identity, and the spaces of Riemannian metrics on $M$ and on $N$. There are several actions of the groups. The space of immersions modulo the group of reparametrizations is an orbifold (with finite isotropy groups). There are several natural weak Riemannian metric of the spaces of metrics. Fixing a Riemannian metric $g$ of bounded geometry on $N$ induces several weak Riemannian metrics on the spaces of embeddings and of immersions and on the diffeomorphism group of $N$. I will review their interplays, their geodesic equations, what is known about well-posedness for them, and what is known about curvatures.](mailto:gauthier@univ-tln.fr)](mailto:gauthier@univ-tln.fr)](mailto:gauthier@univ-tln.fr)](mailto:gauthier@univ-tln.fr)

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[[[[**Title:**](mailto:peter.michor@univie.ac.at)  ***Modeling dMRI data: An introduction from a statistical viewpoint***](mailto:peter.michor@univie.ac.at)](mailto:gauthier@univ-tln.fr)](mailto:gauthier@univ-tln.fr)

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[[[[**Abstract:**](mailto:peter.michor@univie.ac.at)I'll shortly discuss the data generation process, data pre-proccessing and the resulting statistical properties of dMRI data. The main part will concentrate on models and parameter estimation. This will cover the classical diffusion tensor model, different tensor mixtures models and approximations of the orientation density function. I'll provide characteristics of these models, and discuss their interpretation. Examples will be provided based on real data. I'll then reconsider problems from clinical practice and neuroscience research.](mailto:peter.michor@univie.ac.at)](mailto:peter.michor@univie.ac.at)](mailto:peter.michor@univie.ac.at)

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[[[[**Title: *Position-orientation adaptive smoothing (POAS) diffusion weighted imaging data***](mailto:polzehl@wias-berlin.de)](mailto:polzehl@wias-berlin.de)](mailto:polzehl@wias-berlin.de)](mailto:peter.michor@univie.ac.at)

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[[[[**Abstract:**](mailto:polzehl@wias-berlin.de)[Among the obstacles in the analysis of diffusion weighted imaging (dMRI) data noise and a low signal-to-noise ratio (SNR) play an important role especially at the edge of possible spatial resolution. Any noise reduction, however, has to face the fact, that the structures of interest in dMRI are found at multiple scales with sharp discontinuities at tissue borders. Simple non-adaptive filtering procedures for noise reduction will inevitably introduce a significant bias at these borders and will not account for the local properties of the data. In this talk we will introduce a position-orientation adaptive smoothing (POAS) algorithm for dMRI data, that uses the geometric properties of the underlying measurement space, to reduce the noise locally adaptive. We will show, using simulations and experimental data at 7T, that POAS is able to reduce noise without blurring at structural borders. We will refer to some theoretical properties of the propagation-separation approach POAS is based on.](mailto:polzehl@wias-berlin.de)](mailto:polzehl@wias-berlin.de)](mailto:polzehl@wias-berlin.de)](mailto:polzehl@wias-berlin.de)

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[[[[**Title:  *Hopf fibration: geodesics and distances***](mailto:karsten.tabelow@wias-berlin.de)](mailto:karsten.tabelow@wias-berlin.de)](mailto:polzehl@wias-berlin.de)](mailto:polzehl@wias-berlin.de)

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[[[[**Abstract:**](mailto:karsten.tabelow@wias-berlin.de)[We study sub-Riemannian geometry, and in particular, geodesics connecting two given points on odd-dimensional spheres respecting the Hopf fibration. This geodesic boundary value problem is completely solved in the case of 3-dimensional sphere and some partial results are obtained in the general case. The Carnot-Carathéodory distance is calculated. We also present some motivations related to quantum mechanics. Joint work with Irina Markina and Der-Chen Chang.](mailto:karsten.tabelow@wias-berlin.de)](mailto:karsten.tabelow@wias-berlin.de)](mailto:karsten.tabelow@wias-berlin.de)](mailto:karsten.tabelow@wias-berlin.de)

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