Waves in perfect lenses

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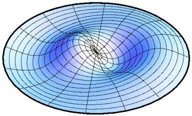
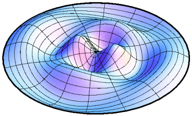
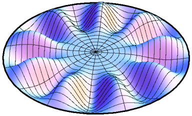
Workshop Třešť, 19 October 2012

Outline

- Perfect lenses and absolute instruments
- Pulses in Maxwell's fish eye
- Gouy phase
- The effect of Gouy phase on pulses
- Spectra of perfect lenses

Absolute instruments and perfect lenses

Absolute instrument (AI) – images a 3D region sharply

B is a sharp (stigmatic) image of A – all rays emitted from A to a non-zero solid angle come to B

Theory of AI within geometrical optics:

[J. C. Maxwell, Camb. Dublin Math. J. 8, 188 (1854)]

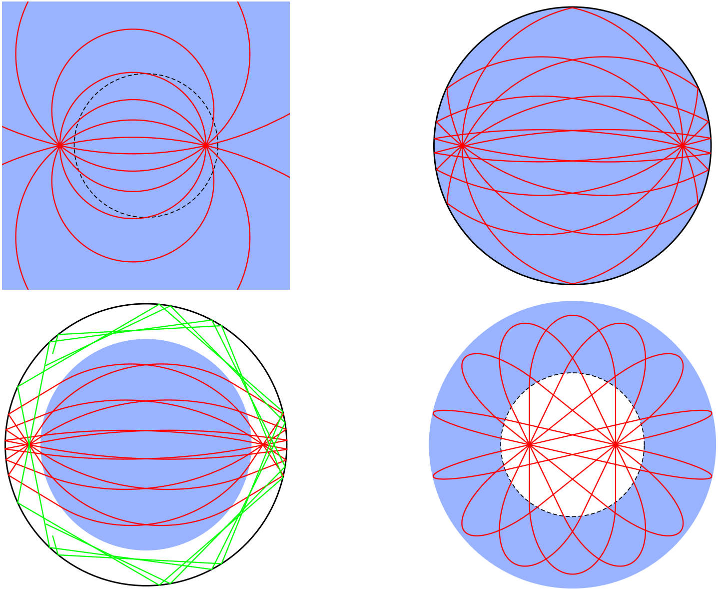
[M. Born & E. Wolf, Principles of Optics]

[J. C. Miñano, Opt. Express 14, 9627 (2006)]

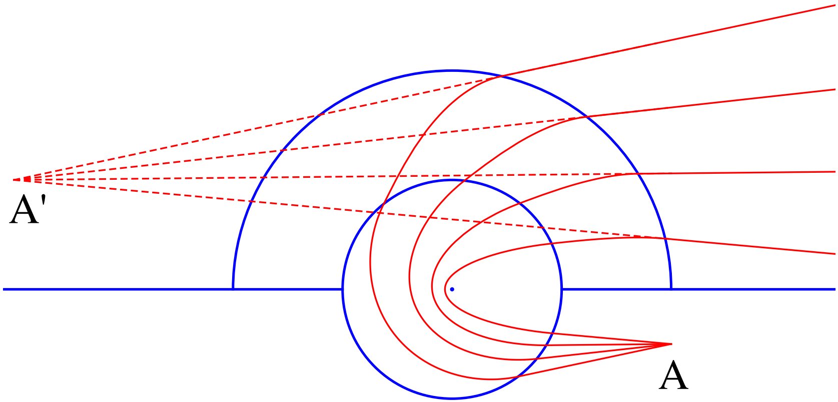
[T. Tyc, L. Herzánová, M. Šarbort, K. Bering, New J. Phys. 13, 115004 (2011)]

Magnifying AI:
[T. Tyc, Phys. Rev. A 84, 031801(R) (2011)]

Examples of absolute instruments



Magnifying absolute instrument



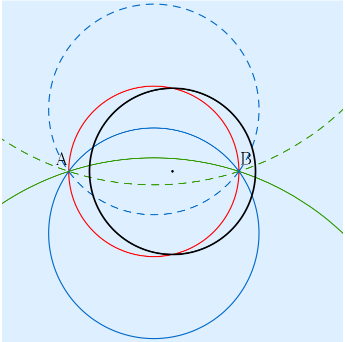
Maxwell's fish eye

Maxwell's fish eye – discovered by J. C. Maxwell in 1854

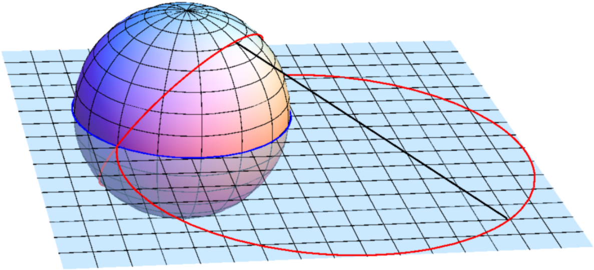
Spherically symmetric refractive index

$$n = \frac{2}{1 + r^2/a^2}$$

Ray trajectories are circles, every point of space has a sharp image



K. Luneburg's explanation (1944) – stereographic projection



Conformal map, magnification such that optical path element in the plane equals geometrical path element on the sphere:

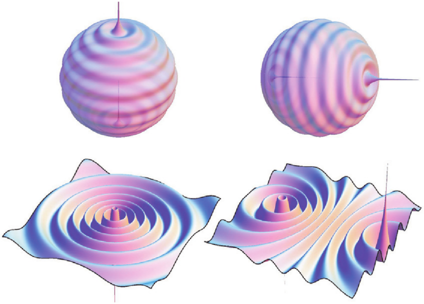
$$n(r)\,\mathrm{d}l = \mathrm{d}L$$

Monochromatic waves in Maxwell's fish eye

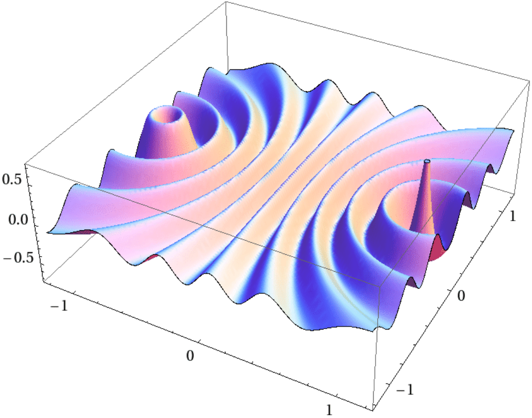
Green's function in 2D:

$$P_\nu = \frac{P_\nu(\cos\theta) - e^{i\nu\pi} P_\nu(-\cos\theta)}{4\sin(\nu\pi)}$$

[U. Leonhardt, New J. Phys. 11, 093040 (2009)]



Super-resolution



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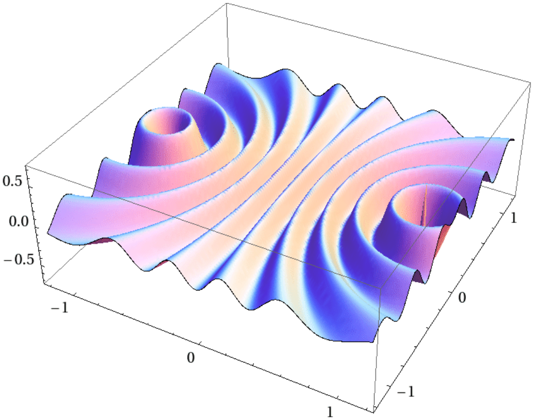

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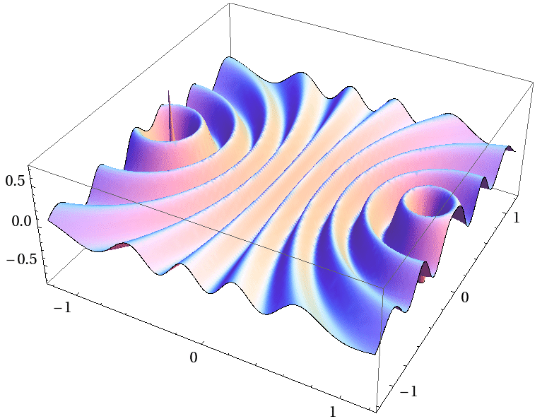
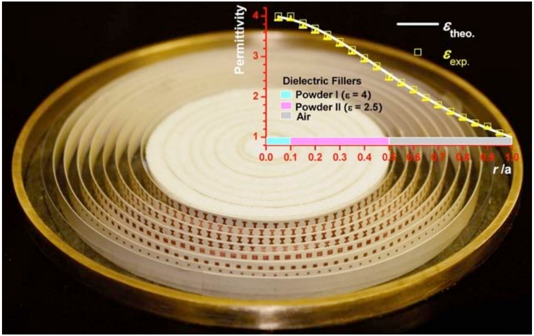
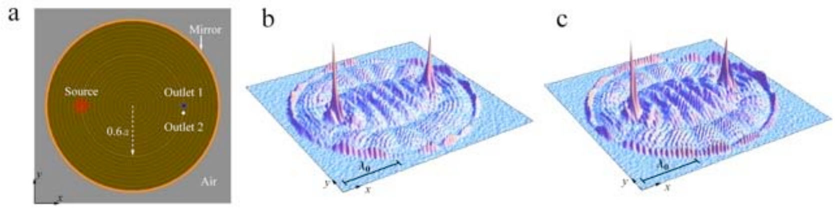


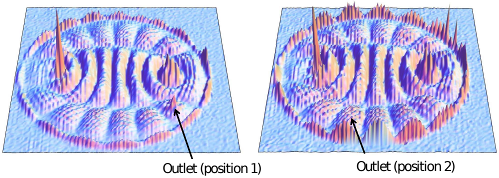
Experiment with microwaves



Radius 5 cm, thickness 5 mm, $\lambda = 3$ cm

Results:





[Y. G. Ma, S. Sahebdivan, C. K. Ong, T. Tyc, U. Leonhardt, New J. Phys. 13, 033016 (2011)]

The role of the outlet

Without the outlet, the convergent wave changes into a divergent wave at the image:

$$\frac{\mathrm{e}^{-\mathrm{i}kr-\mathrm{i}\omega t}}{r} \quad \rightarrow \quad \frac{\mathrm{e}^{\mathrm{i}kr-\mathrm{i}\omega t}}{r}$$

Neither of them satisfies wave equation at the image point, both have a singularity there

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A superposition satisfying wave equation also at the image point:

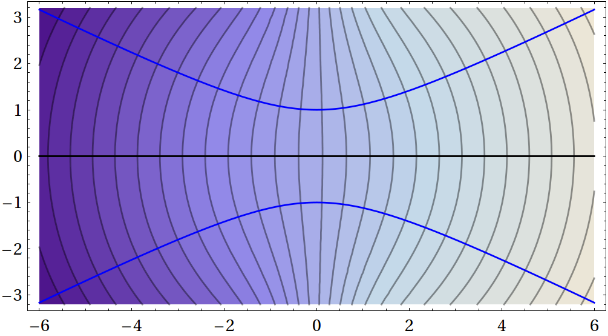
$$\psi(r) = \left(\frac{\mathrm{e}^{-\mathrm{i}kr}}{r} - \frac{\mathrm{e}^{\mathrm{i}kr}}{r}\right) \mathrm{e}^{-\mathrm{i}\omega t} = -2\mathrm{i}k\mathrm{e}^{-\mathrm{i}\omega t}\,\mathrm{sinc}\,kr$$

The diverging wave destroys subwavelength image by interference

Gouy phase

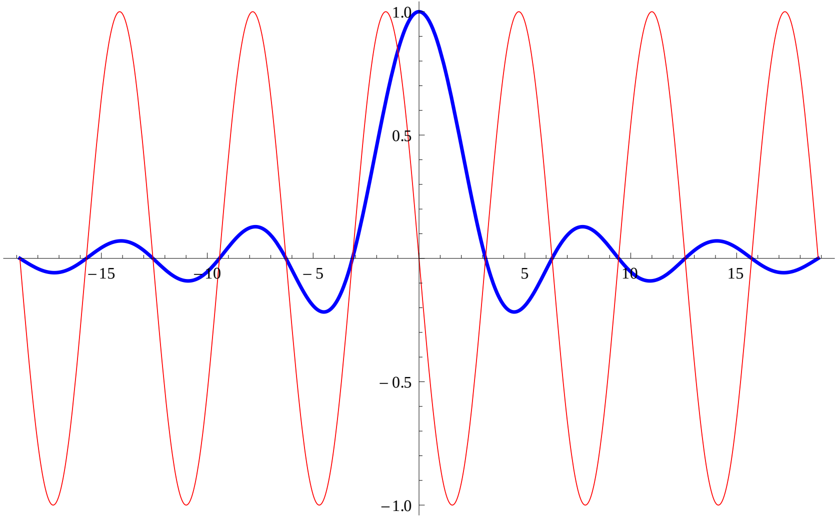
[L.G. Gouy, CR Hebdomadaires Séances Acad. Sci. 110, 1251 (1890)]

Around the focus of a lens (or a waist of a Gaussian beam), there is a phase shift of $-\pi$, wavelength is slightly larger there



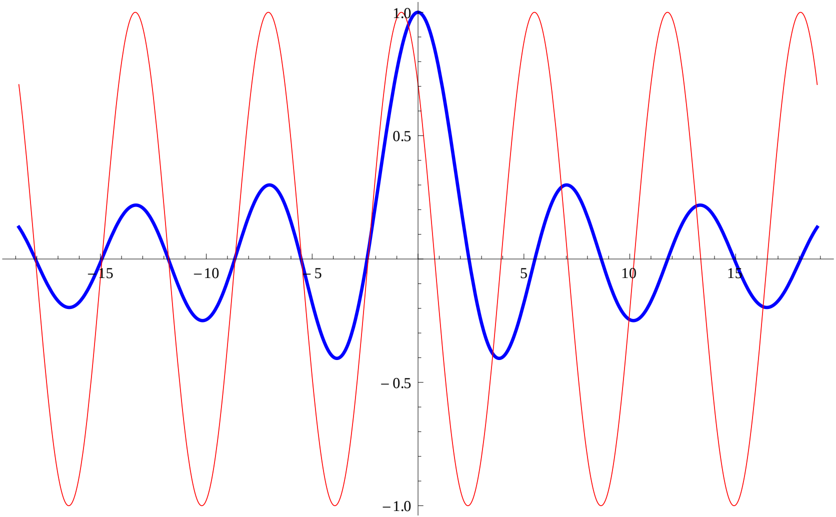
Reason: near the focus, the transversal component of $\vec{k}$ has a larger uncertainty $\rightarrow$ less room for longitudinal component

Gouy phase for spherical waves



Phase change of $-\pi$ in sinc when going through the focus!

Gouy phase for cylindrical waves



Phase change in Bessel function $J_0(kr)$ is $-\pi/2$

Pulses in Maxwell's fish eye

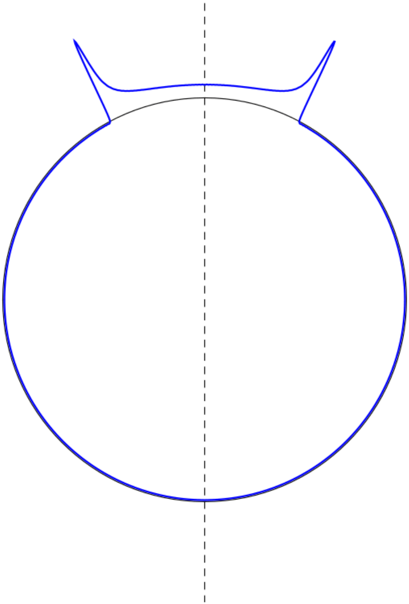
Solution of non-homogeneous wave equation – time-dependent Green's function:

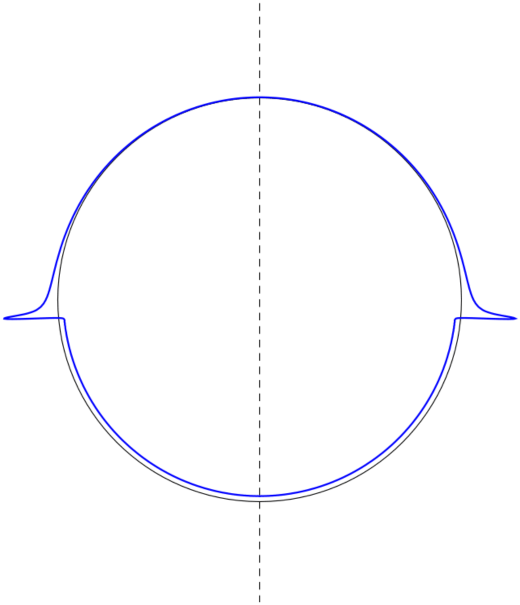
$$\frac{\partial^2 u}{\partial t^2} - \Delta u = 4\pi\delta(\theta)\delta(t)$$

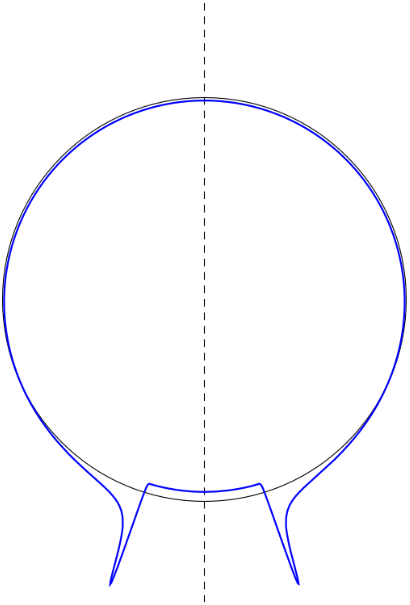
It is a Fourier transform of monochromatic Green's function,

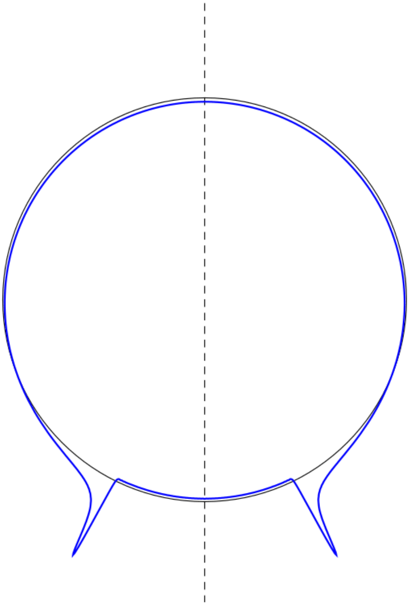
$$G(\theta, t) \propto \Theta(t) \sum_{n=1}^{\infty} \frac{n + 1/2}{\sqrt{n(n+1)}} P_n(\cos\theta) \sin(\sqrt{n(n+1)}\, t)$$

A superposition of spherical harmonics $Y_{lm}(\phi, \theta)$ with $m = 0$









Green's function in 3D & 2D

In 3D, causal Green's function of the wave equation

$$\frac{\partial^2 u}{\partial t^2} - \Delta u = 4\pi\delta(\vec{r})\delta(t)$$

is

$$G(\vec{r}, t) = \Theta(t)\,\frac{\delta(|\vec{r}| - t)}{r}$$

In 2D, Green's function can be obtained from the 3D one using a line source,

$$G(\vec{r}, t) = \begin{cases} \frac{2}{\sqrt{t^2 - r^2}} & r < t \\ 0 & r > t \end{cases}$$

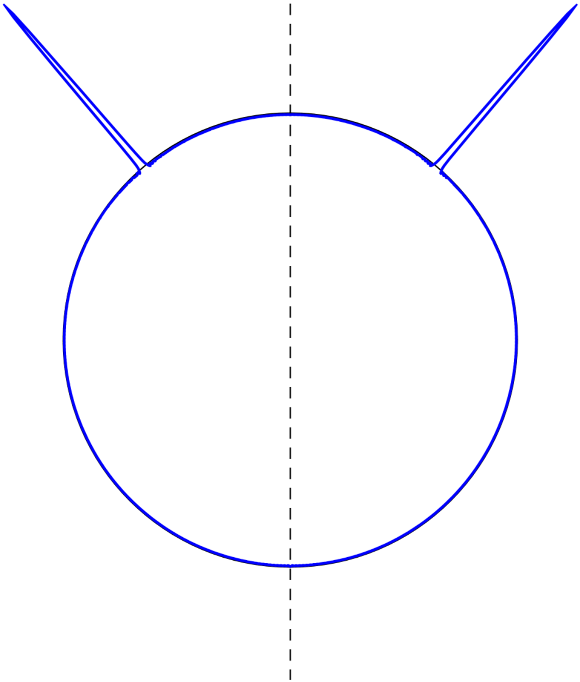
General feature of Green's functions in 2D – wake field following the pulse edge (happens in even dimensions)

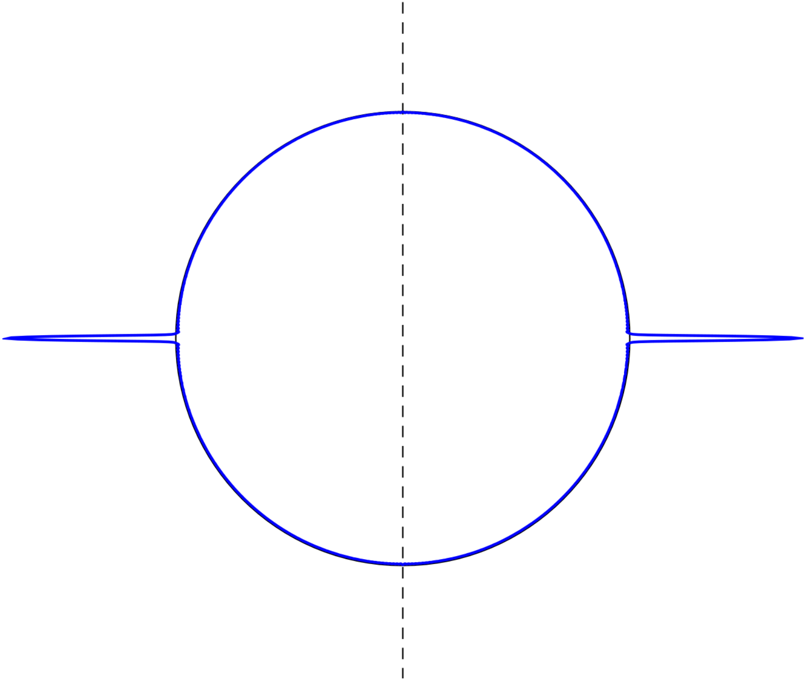
δ-pulses

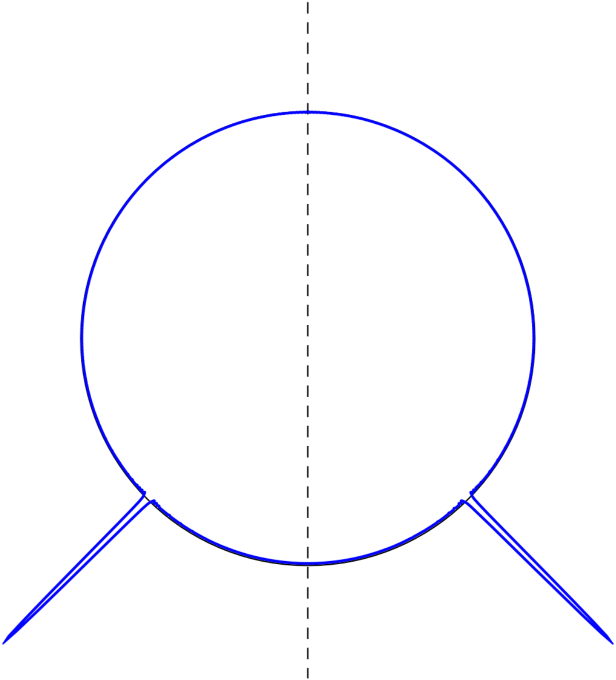
It is impossible to extract the pulse energy at the image by applying a pulse at a single moment of time

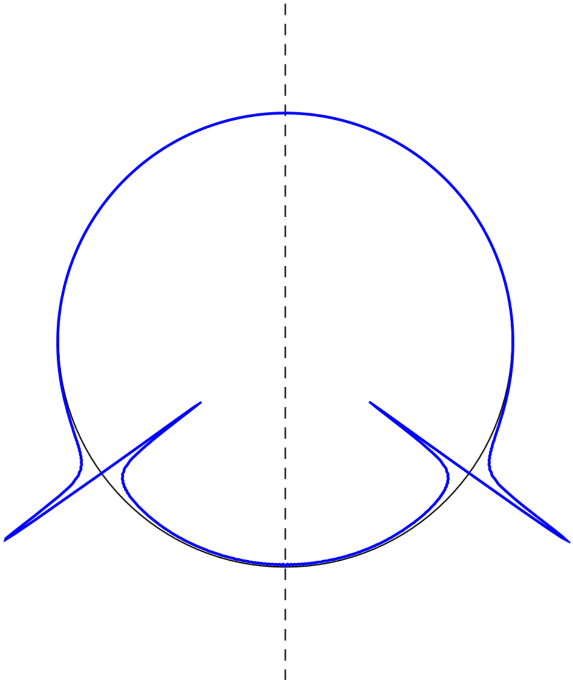
Using asymptotic formulas for Legendre polynomials, one can create sharper pulses than Green's function

Dirac delta-function-like pulse









Dirac δ-pulse in 2D free space

A pulse

$$\psi(r,t) = \sum_{n=1}^{\infty} \sqrt{n}\, J_0(k_n r) \sin\left(k_n t + \frac{\pi}{4}\right)$$

Using the asymptotic form of the Bessel function,

$$J_0(x) \approx \sqrt{\frac{2}{\pi x}} \cos\left(x - \frac{\pi}{4}\right)$$

we get

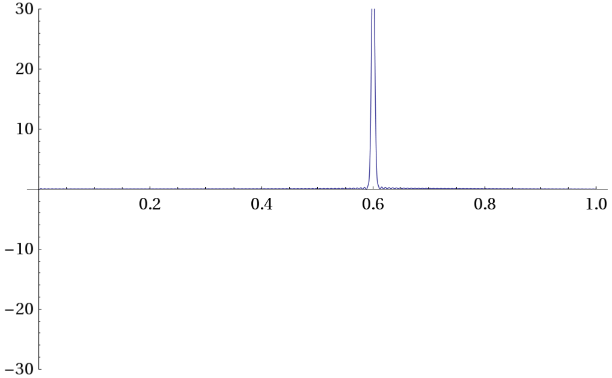
$$\psi(r,t) \approx \sqrt{\frac{1}{8\pi r}} \sum_{n=1}^{\infty} \left[\left(\mathrm{e}^{\mathrm{i}\pi/2}\mathrm{e}^{-\mathrm{i}k_n r} + \mathrm{e}^{\mathrm{i}k_n r}\right)\mathrm{e}^{-\mathrm{i}k_n t} + \mathrm{c.c.}\right]$$

Again Gouy phase!

[T. Tyc, Opt. Lett. 37, 924 (2012)]

$$\psi(r,t) \approx \sqrt{\frac{1}{2\pi r}} \sum_{n=1}^{\infty} \{\cos[k_n(r-t)] + \sin[k_n(r+t)]\}$$

$0 < t < 1$



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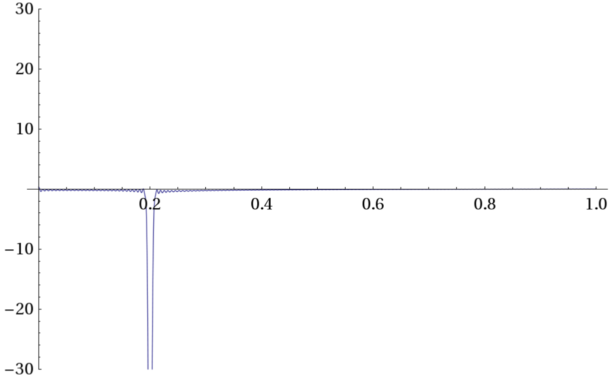
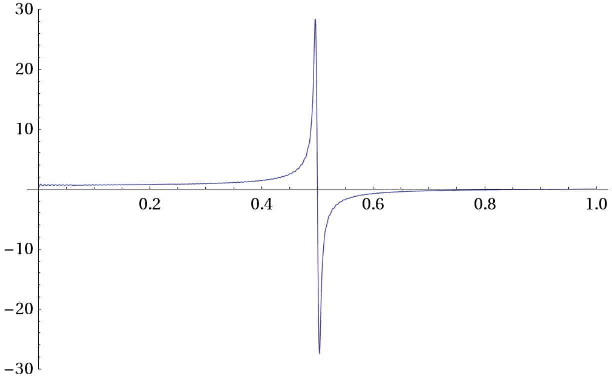
$1 < t < 2$



$$\psi(r,t) \approx \sqrt{\frac{1}{2\pi r}} \sum_{n=1}^{\infty} \{\cos[k_n(r-t)] + \sin[k_n(r+t)]\}$$

$2 < t < 3$



Spectra of absolute instruments

We represent the spectrum by a function $\omega(\nu)$ indexing the eigenfrequencies

For $\nu \in \mathbb{N}$ we put $\omega(\nu) = \omega_\nu$, for $\nu \notin \mathbb{N}$ we define $\omega(\nu)$ by interpolation

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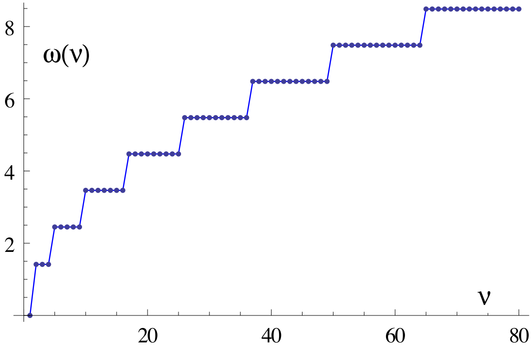
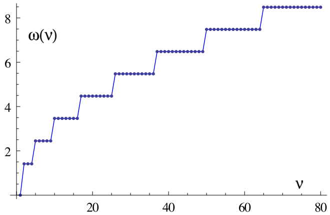
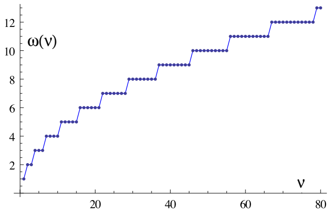
For Maxwell's fish eye:



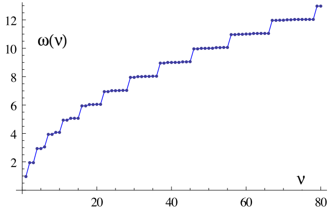
Comparing spectra of AI and non-AI



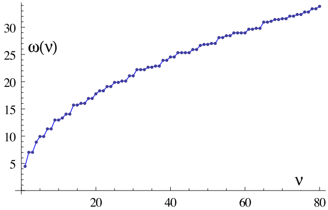
Maxwell's fish eye



Hooke profile



Miñano lens



square cavity

Conclusion

- Waves in AI have distinct properties
- Maxwell's fish eye can provide super-resolution
- Without the outlet, the sharp image is blurred by Gouy phase
- Gouy phase has also a dramatic effect on light pulses
- Spectra of AI have a very distinct character
- Still many things to discover