Algebraic Methods in Quantum Logic (AMeQL)

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Elements of monoidal topology

Sergejs Solovjovs

Department of Mathematics and Statistics
Faculty of Science, Masaryk University
Kotlarska 2, 611 37 Brno, Czech Republic
solovjovs@math.muni.cz

In this lecture course, we will outline briefly the theory of *monoidal topology*, which is an approach to categorical topology based in monads and quantales. Motivated by the representation of topological spaces as lax relational algebras with respect to the ultrafilter monad on the category **Set** of sets, the theory replaces the ultrafilter monad with an arbitrary **Set**-monad $\mathbb{T}$, and the two-element chain with a unital quantale V, thereby obtaining the construct $(\mathbb{T},V)$-**Cat** of $(\mathbb{T},V)$-*categories* and $(\mathbb{T},V)$-*functors*. Particular, examples of the constructs of the form $(\mathbb{T},V)$-**Cat** are the category **Top** of topological space, **Ord** of preordered sets, **Met** of metric spaces, **ProbMet** of probabilistic metric spaces, **Cls** of closure spaces, and **App** of approach spaces. Moreover, the constructs $(\mathbb{T},V)$-**Cat** have convenient properties, e.g., are topological.

During the lectures, we will introduce the constructs $(\mathbb{T},V)$-**Cat** in full detail (recalling the necessary preliminary background on monads and quantales); show how to incorporate the above-mentioned examples in their framework; describe basic properties of the constructs $(\mathbb{T},V)$-**Cat** (e.g., provide the explicit form of their corresponding initial structures, as well as show how to get functors $(\mathbb{T}_1,V)$-**Cat** $\to (\mathbb{T}_2,V)$-**Cat** and $(\mathbb{T},V_1)$-**Cat** $\to (\mathbb{T},V_2)$-**Cat** for different monads and different quantales, respectively); and consider one explicit example of the application of the theory of $(\mathbb{T},V)$-categories to general topology, i.e., a particular generalization of the Kuratowski-Mrówka theorem on the equivalence between the concepts of proper (stably closed) and perfect map.