Teacher Training for Teaching Learning Disabled Individuals - Dyscalculia

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1. Learning disabilities and teacher training

Some children show problems with reading, writing, spelling or mathematics when attending school – it is about 4 – 6 % of the population - although they are of average or even higher intelligence and they have sufficient support from their families and schools. The causes can be various – they can be connected to mild brain dysfunction, they can be genetic, they can be caused by influences from a child's early stages of development, and so on. There exist many theories which attempt to reveal the causes of specific learning disabilities and they offer many solutions. Some are based on brain disorders, some on insufficient function of analysers (seeing, hearing), others on disturbed communication between the child and the world.

The most common developmental learning disabilities are:

Dyslexia – reading disability, it affects especially the speed and correctness of reading, or understanding of the text read.
Dysgraphia – writing disability which affects legibility of writing, acquiring single letters and comprehending the relation sound-letter.
Dysortographia – spelling disability with specific mistakes, e. g. distinguishing hard/soft syllables, short/long vowels, sibilants. This disability depends on the nature and specific features of a language, it does not appear in all languages, it can be found in Czech.
Dyscalculia – mathematical disability which affects mathematical concepts, number operations, space visualisations, etc.
Dysmusia – musical skillsdisability.
Dyspinxia – drawing abilities disorder.
Dyspraxia – coordination disability.

We must, however, point out that there exist also various other types of disabilities: concentration disorders, left/right orientation disorders, space orientation disorders, speech disorders, hearing disorders, seeing disorders, fine and gros motor skills disorders, as well as behaviour disorders resulting from learning disorders. All these disorders influence one another and they may result in weakening functions which are necessary for developing educational skills and abilities.

Teaching learning disabled individuals needs a special preparation. Teachers should be aware of the disabilities and should go through a special instruction to know help such persons. Teacher training, for both future teachers and teachers who already teach, employs both theoretical knowledge and concrete individual work with people having learning disabilities. The following areas should be included in that training:

1. Knowledge about developmental learning disabilities, their symptoms, and influences.
2. Diagnostics and analysis of specific problems of an individual, observable manifestations of a disability, possible causes.

3. Preparing an individual educational plan.
4. Preparing re-educational exercises.
5. Using various educational strategies, constructivist approaches are preferred.

Masaryk University offers teacher training in both pregradual and postgradual study programmes. The pregradual preparation concentrates more on theoretical level although many students work with learning disabled children and the results of such work are included e.g. in their Bachelor theses. Postgradual students gain specific experience from work with children in a class and they require concrete answers to their problems.

A crucial part of the preparation of teachers for children having learning disabilities is a great emphasis on individual work – a teacher must always have in mind that such children need empathy, understanding, patience, positive motivation, individual approach, and special methods.

2. Dyscalculia – diagnosis and analysis causes and problems

We identified mathematics learning disability as dyscalculia, however, a person's success in mathematics is to a certain extend influenced by any learning disability. E. g. dyslexia or dysgraphia can significantly affect an individual's results in maths. Even though we shall concentrate on dyscalculia in the following, it is important to bear in mind that learning disabilities often occur in combination.

Dyscalculia involves having difficulty in the area of building the most elementary concepts of mathematical skills and using them in higher levels. Some typical manifestations of this disability are the following examples:

- Comprehension of the concept of natural number (later of integer, fraction, decimal number, negative number, real number).
- Reading and denoting numbers – a child has problems with distinguishing numbers of a similar shape, with denoting and reading multi-digit numbers.
- Problems with comprehending mental and written algorithms, doing more operations in one exercise.
- Problems with applying operations to practical exercises.
- Problems with measurement units and their conversions.
- Problems with distinguishing geometrical shapes, space location of objects, drawing geometric shapes.

On the formal level we distinguish six types of dyscalculia:

- practognostic (difficulty with manipulation of objects),
- verbal,
- lexical,
- graphical,
- operational (difficulty understanding operations),
- ideognostical (difficulty understanding mathematical relations).

The basic criteria which help to identify a child as dyscalculic can be presented as the following:

- There exists a clear disproportion between the intelligence of the child and his/her results in mathematics.
- The child's intellectual powers are not below average.
- The child's problems are not caused by an illness or of social emotional nature.
- The child lives in a normal family environment which provides positive motivation.
- Medical examination shows a dysfunction of central nerve system and of cognitive brain centres.

Apart from specific developmental learning disabilities, there are other factors influencing a child's success in mathematics. These are especially disabilities caused by the child's individuality – e. g. their age immaturity for a specific subject-matter (they will easily understand the matter in six months, or a year), their volitional qualities (inability to make themselves work systematically, which mathematics needs, laziness), self-confidence, anxiety, various psychological barriers, such as a fear of mathematics or some of its topics, a fear of tests or being examined. A loss of hopes of being successful and being an outsider in the class also affect the child's results. Disabilities which occur in childhood last in some way in the adulthood.

Another group of potential problems relates to the personality of the teacher and the way of his/her teaching. An insufficient professional competence, violation of relationships in mathematical language usage, problems in communication with pupils, formalism at work, impatience, problems with evaluation and classification belong to the causes of little success in the pedagogical work as far as the success of their pupils in the subject is concerned. The anticipation of poor results of a learning disabled child is not motivating for the pupil either.

A mathematics teacher has to take into consideration specific features of the subject, which involve high abstraction of the concepts, generalizing, proving. Mathematics is also a subject in which every element of a higher level supposes precise knowledge and comprehension of elements and skills of lower levels. The matter systematically continues, and if a child does not master one area, they have difficulties carrying on. In that case there is only one solution – to return to the matter which is a primer cause of the problem. This, however, requires a high professional and methodical erudition of a teacher educating learning disabled pupils.

The parents' attitude to a child having learning disabilities must not be neglected either. The work with parents is sometimes more difficult than the work with children. Only a certain number of parents try to understand the child and seek professional help of pedagogic and psychological counsel, and they adopt the child's schooling to their disability. Unfortunately, there are often inadequately ambitious parents who do not show enough patience which a dyscalculic child needs – they overburden the child with too much learning, many-hour preparation for school, or extra one-to-one teaching, they do not respect the child's getting tired soon. Another group of parents give up, they leave the child without help (they say: "There is nothing we can do, we were no good at maths either."). On the other hand, others try to improve the child's work, they themselves provide the child with extra exercises and make up their own methods which may, however, prove to be unsuitable in the future.

Based on a survey (usually semi-structured interviews) of adult students who suffered a learning disability as children, some remarkable facts arose:

- Mathematics was a subjects of formulas and theorems which I didn't understand.

- If I am supposed to calculate something, I am afraid I won't be able to do it on my own.
- As dyslectic I always had problems with reading instructions and if the teacher didn't help me, I wasn't able to calculate anything.
- I have problems with imagination, it took me very long to understand a picture, e.g. a net of a cube.
- Mathematics was torment for me, I went to mathematics lessons with fear.
- Mathematics lessons depended on the work and understanding of the teacher.
- Being in a special class for children with learning disabilities led to giving up, we didn't have a sufficient motivation to obtain better results.
- I suppose that my parents' resignation (you haven't got a flair for mathematics, you will never understand or learn it) prevented me from understanding it and from learning its beauty.
- I am afraid that my children will inherit my disability, I don't want them to have to go through the horrors, and misunderstanding, sometimes humiliation or ridiculing that I met.

3. Common dyscalculia deficits

A dyscalculic person usually has problems with mastering basic mathematical facts and skills by the traditional methods of teaching, therefore a systematic special instruction is essential for a successful work with such individuals. The remedial teaching has to start at the lowest point which the person does not comprehend. Areas where dyscalculia shows itself are elementary mathematical concepts of number and numeric operations.

Understanding the notion of a natural number is preceded by many activities which a child meets in an early childhood (2 – 3 years) and which results in comprehending classes of equivalent groups of elements and in a natural number. This means especially looking for common properties, sorting out, assorting, classifying, creating groups of the same number of elements and so on, when a child does not count, but perceives quantity. It is only later when they denote the number of elements of a group by a number, they learn to write and read numbers. If a child who has a predisposition for a developmental learning disability does not go through this process he/she does not have a possibility to comprehend the notion of number. An analogical process happens with every further enlarging of a number field – natural numbers through a hundred, a thousand, a million, ... as well as rational and real numbers (decimals, fractions, powers, roots).

Understanding each numeric operation is also a complicated process and mere memorizing of some procedures is not a solution. Every phase of the process of building mathematical concepts has its specific features and it is only an individual diagnosis which may reveal the problems of every child with maths learning disability. Because of this considerable particularity it is not possible to state all problems, nevertheless, we shall mention the most frequently occurred ones:

- if a child does not have enough stimuli in the pre-school age, they cannot create a group of a given number of elements, they cannot express the number of elements in a given group,
- a child has problems with sequencing numbers – they count e. g. one, two, three, four, seven, five, four, ... later they count thirty eight, thirty nine, thirty ten,

- a child is not able to compare numbers, they confuse comparing a number of elements with the size of objects, they cannot use the symbols of comparison “<”, “>”,
- a child has problems with distinguishing numbers of a similar shape (e. g. 6 and 9),
- a child has problems with denoting multi-digit numbers (they do not distinguish e. g. 48 and 84, later 342, 324),
- a child does not understand the meaning of the positional decimal system, e. g. they write what they hear – they write down three hundred and twenty as 30020,
- a child has problems with showing numbers on the number line,
- a child does not understand the meaning of rounding numbers,
- a child does not understand the reason and meaning of operations, especially the symbols “-“ and “÷”,
- a child confuses an operation with denoting numbers, e. g. 1 + 3 = 13, 24 + 62 = 2462,
- a child does not master mental calculations, the biggest problem is subtraction with borrowing tens – they subtract 62 – 27 as 60 – 20, 7 – 2,
- a child is not able to memorize algorithms for numeric operations – they either concentrate on connections, and then make mistakes in notation, or they concentrate on notation and make mistakes in connections.
- they have problems with measurement units and their conversion,
- they are not able to solve practical or applied exercises because they do not know which operation to use.

4. Reeducation of dyscalculia

When working with maths learning disabled persons, it is necessary to bear in mind that everyone is a strong personality and thus needs an individual approach. What is successful with one individual need not be beneficial to another one, thus it is not possible to recommend any general intervention. We can, however, summarize general reeducational rules which should be always followed when dealing with dyscalculic individuals. These are known as “Ten Commandments”:

1. **Specifying diagnosis** – stating the main problems concerning mathematics; in which areas the individual has problems, what are their causes, what the individual’s relationship to mathematics is.
2. **Respecting logical structure of mathematics and its specific features** – in mathematics the comprehension and mastering of every element of lower level are necessary conditions for mastering elements of a higher level. Reeducational exercises thus has to begin in the subject-matter which the individual stopped understanding. The procedures must respect mathematical laws and it must be possible to use them also in other subject-matter.
3. **Comprehension of basic concepts and operations** – all fundamental concepts have to be generated on concrete models and all operations with numbers must be deduced from manipulative and mental activities of the individual. At the same time it is necessary to use various forms of work and think of many new situations.
4. **Create the “I see” effect** – the individual comes to a piece of knowledge on their own “oh, I get it now” and accepts the knowledge as their own. We must remember that we can transfer information, but not understanding.
5. **Using all senses** – involving as many senses as possible for gaining mathematical knowledge: sight, touch, hearing, moving, so that the individual feels comfortable and the senses contribute to gradual overcoming the problems. Suitable games play a significant role.

6. **Discussions** – “what do you see?” – to find out whether the individual can see what the teacher does. Every person has their own communicative ways by which they gather knowledge, the teacher has to find them in discussions. There is no mathematical blindness, everyone can find their own way to mathematics. Dyscalculia does not allow one to give up and do nothing.
7. **Mastering of the subject-matter by heart** – to an extend the individual is able to and it is meaningful, mathematical subject-matter cannot be based on memory only, without understanding and correct deducing. We have to find a suitable balance between deducing and drill.
8. **Increasing demands for the child’s independence and activity** – the individual can create their own materials, examples and aids, or they can participate in making them, the person can realize their own deficits and take part in their overcoming in an interesting and active way. The teacher can make use of project teaching.
9. **Constant encouragement** – the individual needs positive experiences, ease, praise, cheerful way of reeducational exercises, game therapy, no overburdening and at the same time no idling. The individual should be praised after every success, no matter how little it is.
10. **Work according to an individual plan** – we prepare a plan according to the needs of every individual. One-to-one teaching, or individualized teaching in a class.

5. Assessment of learning disabled individuals

By assessment of a person we mean every teacher’s reaction to the individual – verbal as well as nonverbal. Every pupil or student expects teacher’s comments to their work because the work has been done, no matter what the result is. The teacher, therefore, has to assess each learning disabled person individually, we cannot compare them with their classmates. Persons having a specific learning disability are usually of average, or higher intelligence, therefore we should not mistake problems resulting from the disability for laziness, or incapability. Our assessment should concern every little success during an activity, we should encourage the pupil by positive environment (praise, smile, etc.).

When assessing dyscalculic individuals we must assess what the person can do, not what they cannot. From a wide range of work which influence the assessment, or classification we choose those which are favourable to them:

- as for the oral or written form, we choose the one in which the individual expresses themselves more easily and better,
- in written work, we check the whole method of solution in great detail, we evaluate the train of thought, not only the result,
- we assess the quality of work (regarding the individual’s train of thought, and effort), not the quantity,
- we set a reasonable task (as far as both the content and time limit are concern) depending on the individual’s possibilities,
- we prepare instructions of a task in a suitable way (taking into account the individual’s disability – dyslexia, dysgraphia) – e. g. pre-printed on a work sheet, using pictures,
- first we begin with several tasks which they already know how to solve, then we teach some new elements (basing the new elements on the known tasks),
- we ensure the optimal environment for work – the person must feel at ease,
- every work of the individual is used as feedback for both the pupil/student and the teacher – we analyse and correct mistakes together with the person, the teacher makes

an analysis leading to understanding the individual's train of thought and prepares further methodical plan.

6. Mathematics and the social status of a person

The social status of a person is influenced by their development in the childhood and their relation to mathematics can play a certain role as well. If an individual does not feel successful in mathematics, they, naturally, try to avoid this subject when deciding about the future career. They choose one in which they do not meet mathematics much (e. g. arts or humanities), or where their developmental disabilities need not affect them e. g. in natural sciences. Many notable personalities had problems with mathematics in their childhood, nevertheless they achieved remarkable results later, some of them even in mathematics or physics. Let us recall some famous names:

- Albert Einstein hardly passed the subjects, he had great difficulty with reading.

- Thomas Alva Edison belong to bad pupils, he never mastered skills like writing, spelling, and even arithmetic.

- The physicist George Gamov is described in *My World Line* by his student, a famous astronomer, Vera Rubin in the following way: "He could not write or count. It would take him a while to tell you how much is 7 times 8. However, his mind was able to comprehend the universe."

- Mathematician N. N. Luzin belongs to people with a slow reaction. He also developed slowly, he did not succeed in school, especially in mathematics.

- David Hilbert, one of the greatest mathematicians of 20th century, made an impression of a dull person who slowly and with difficulties understands what other people say.

We could mention far more examples when a seemingly "dull" pupil who has problems at school shows to be a genius in the future. It is therefore necessary to treat learning disabled children sensitively, try to understand their problems, and look for ways which will facilitate their learning.

A person having a learning disability will find a way how to cope with the problems later as an adult, however, every time when they find themselves in a situation where they need to use skills they have problems with, they realize how much effort they need to solve the situation. Many problems (especially from the area of numeric calculations) are possible to minimize by using compensation aids (calculator, computer), however, we can also encounter situations when those aids are of no use to us. Most people conceal their problems because they might be afraid of social degradation even though, unfortunately, "not knowing mathematics" seems to be socially acceptable.

7. Adult education – teaching mathematics at MU

The theory of adult education deals with methods, forms and technologies which take into account specific features resulting from the situation of adults. It is not possible to apply automatically the methods and forms used for educating children and young people especially because adults are very sensitive to the work and attitude of teachers. They expect professional, tactful and partner approach. When teaching mathematics we have to consider previous experience of adult students with maths education and their relationships to mathematics. We have to help them with overcoming the fear of mathematics and little interest or problems with mathematical concepts and methods of work. It can be difficult at the beginning, however, their experience and persistency usually help them to succeed.

If an adult suffered some learning disabilities as a child, the disabilities will appear in a different form and to a certain extent again. Dyscalculic problems can manifest themselves in working with powers and in probability exercises, and generally in practical exercises. Teachers should take a note of problems of this nature because overcoming them costs trainees a great deal of energy. Reeducation of adult dyslectics must follow the same rules as mentioned above. Especially important is to start at the point which the person does not comprehend.

Adult education is realized in several levels at Masaryk University - education of teachers as a part of life-long education, education of teachers who want to complete their pedagogical education, education of adults who want to complete their secondary school education, general interest mathematics for adults and seniors.

Education of teachers as a part of life-long education

This form of education offers to teachers who graduated from a university and have full pedagogical qualifications a series of courses, lectures, seminars, exercises or practical activities focusing on various topics of school mathematics. Teachers can e. g. get acquainted with new methods of work, various approaches to treating mathematical topics in lessons, with possibilities of implementing individual, individualized, group or project teaching, etc. The courses also include a preparation for teaching pupils with developmental learning disabilities on the one side as well as for teaching gifted pupils on the other side.

The courses are organized in a way so that teachers themselves experience the role of a pupil so that they go through the same process as pupils at school. The most suitable form of work proved to be the constructivist approach when teachers find new information, knowledge, rules, etc. by their own activities. At present we aim new courses to the changes in the curricula – to creating school educational programmes implementing the Frame educational programme.

Education of teachers who want to complete their pedagogical education

This types of education is offered by the Faculty of Pedagogy to teachers who already teach, however, they do not have full pedagogical qualifications. The preparation consists of two parts: subject specific and didactic. Mathematical education include arithmetic and algebra in the first two years, geometry in the third year, and the fifth year is devoted to didactic of mathematics, discrete mathematics, methods of solving exercises, work with pupils of various levels, interest mathematics, etc. Nowadays a distance form of studies and using e-learning courses are preferred.

Education of adults who want to complete their secondary school education

Adult education is specific due to the fact that adults are already participants in the labour market and they come with the aims resulting from their profession. Either they need to master mathematics for secondary graduation examination, or they need specific mathematical knowledge for their profession. Adult students usually come with incoherent mathematical knowledge, they often have deformed ideas about mathematics, including distrust and anxiety, they can be tired because of their demanding jobs. On the other hand they are persistent and ambitious, and they make a great effort to overcome initial difficulties. A teacher must not reduce mathematics to mere facts and doing exercises, they also want explanations and reasoning. They do not accept mechanical learning, the memory is more shifted to logical memory in a later age. They are able to master algorithms quickly, most of the trainees think about problems, they try to find logical explanations. Their experience enables them to solve problem situations, they are interested in applications related to their professions. They learn independently, they are able to use information technologies, and find learning methods which are suitable for them. They cope with possible failures worse than young people, they therefore need encouragement, praise, and feeling that the teacher is their partner. Naturally, even among adult students we can find those who try to achieve maximum with a minimal effort.

General interest mathematics for adults and seniors

The courses of interest mathematics are usually offered in the form of setting interesting exercises which provide participants with impulses to think logically, show them new experiences, and make them use brains actively.

Generally, the forms of work for adult education are chosen in the way so that adults work independently, they can check the work immediately, and the obtained result is interesting. It is suitable to use also topics from combinatorial analysis, statistics, and probability as they can serve for reinforcing both numeric operations and combinatorial and statistical thinking.

9. Case study

We shall present the summery of four cases – the diagnosis and reeducational strategies of individual work with four children: David, Peter, Lucy and Michael.

David

Problem area: David has problems with addition and subtraction of numbers through 20 with regrouping, i. e. he finds exercises similar to 4 + 8, 16 – 9 very difficult.

Analysis of the sub-problems:
1. Practicing decomposition of numbers into two parts.
We used games and physical activities, e. g. for number 6:

- clapping: twice on the right, four times on the left,
- representation by marbles: oo oooo,
- representation by sticks: // ////
- playing the piano: two low tones, four high tones,
- then the decomposition was written: 6
  2 4

2. Practicing addition with regrouping.
We started with the exercise 7 + 8

3 5

thus we counted: 7 + 3 = 10, 10 + 5 = 15, therefore 7 + 8 = 15.
Discussions with David showed that he had not accepted this model, he had made his own model – decomposition with number 5 (the model being five fingers), he decomposed the numbers in the following way:

7 8

5 2 5 3

He, therefore, counted 5 + 5 = 10, 2 + 3 = 5, 10 + 5 = 15, thus 7 + 8 = 15.
David used this model confidently, he counted without mistakes, therefore he hold on to the model he had arrived at on his own.

3. Practising subtraction with regrouping.
Situation similar to addition repeated. We started deducing exercises of the type 16 – 9 by the most common way: 16 - 9

6 3

i. e. the subtrahend is decomposed in such a way that we subtract the units of the minuend, thus we count 16 – 6 = 10, 10 – 3 = 7, therefore 16 – 9 = 7.

David did not accept this model either, and he created his own method – he decomposed the minuend to a ten and units: 16 - 9

10 6

and counted 10 – 9 = 1, 6 + 1 = 7, thus 16 – 9 = 7.
Practising with other exercises showed that David used his method reliably and without making mistakes.

Peter

Problem area: Peter has problems with written algorithms – addition, subtraction – he does not comprehend carrying and borrowing tens. E. g. in the example

$$\begin{array}{r} 1\,278 \\ +\underline{\ \ 569} \end{array}$$

he makes mistakes of the following type:

a) he counts 9 + 8, and writes down 17,
b) he does not add one ten, but 10, i. e. he counts 9 + 8 = 17, writes down 7, and then adds 10 + 6 + 7,
c) he does not add a ten at all, he counts 9 + 8 = 17, writes down 7, and continues 7 + 6 = 13, writes down 3, ...
d) he is not confident at mental counting with regrouping through 20.

Analysis of the sub-problems:

1. deducing mental adding with regrouping through 20 again – we can use graphical tools (colouring squares in a grid, ...), we must follow the arithmetical model Peter uses (some children count according to the model
8 + 7 = 8 + (2 + 7) = (8 + 2) + 7 = 10 + 5 = 15, i. e. they complete the first addend to ten, however others count
8 = 5 + 3, 7 = 5 + 2, 5 + 5 = 10, 3 + 2 = 5, i. e. they decompose numbers with the use of 5.

2. reinforcing decomposition of numbers to ten and units: 17
10   7

3. reinforcing the knowledge that ten units make one ten.

4. reinforcing the development of numbers in the decimal system: 17 = 1x10 + 7.

5. making the knowledge automatic.

6. practicing estimating results: what will be probably the result.

7. Remedial exercises for mastering the written addition:

a) first adding numbers without carrying tens, e. g.
$$\begin{array}{r} 354 \\ +\ \underline{235} \end{array}$$
b) adding numbers where there is only one carrying, e. g.
$$\begin{array}{r} 6\ 257 \\ +\ \underline{538} \end{array} \qquad \begin{array}{r} 1\ 362 \\ +\ \underline{2\ 573} \end{array}$$

c) adding numbers where we get partial sums 10, e. g.
$$\begin{array}{r} 562 \\ +\ \underline{258} \end{array} \qquad \begin{array}{r} 748 \\ +\ \underline{156} \end{array} \qquad \begin{array}{r} 965 \\ +\ \underline{134} \end{array}$$

d) adding numbers with more carrying.

Work with models:

a) practical applications – examples from the real life in which we need to add big numbers,
b) formal aid – squared paper for denoting numbers to coloured columns strictly according to the orders.

Compensation tool: a calculator – only if it is used functionally, i.e. if the child can work with it and can estimate results.

Lucy

Problem area: Lucy has problems with calculations with negative numbers, she does not comprehend the symbol „minus“ „-„.
She does not understands why -2 > -5, she dreads exercises of the type (-5) – (-8).

Analysis of subproblems.

1. Explaining the meaning of „minus“ as:

a) a symbol for the operation of subtraction, e. g. 12 - 7
b) denoting a negative number, e. g. the temperature was – 6°C
c) denoting the opposite number to a given number, e. g. the opposite number to 5 is (-5), the opposite number to –7 is number –(-7) = +7.

2. Understanding the concept of negative numbers.
Lucy was offered the following models:

- physical model – measuring temperature,
- finance model – assets and debts,
- history model – denoting events B. C.
- geographical model – mountains and troughs,
- game model – denoting positive and negative points in a game.

Lucy chose the finance model.

3. Work with whole numbers – number line, comparison

Lucy selected two colours to denote numbers formally – she denoted positive numbers with a blue colour, and negative ones with a red colour.

First she practised reading and denoting numbers. She represented numbers on the number line. With the help of the financial model and the number line, Lucy started to realize the concepts, intuitively she started to comprehend the notion of number and its absolute value, although this concept had not been mentioned. When we continued with comparing numbers, problems arose. Lucy could not understand for a long time why e. g. $-7 < -2$, she fastened too much to the concept of „bigger debts“ and to the distance of the image of a number on the numbers line from the image of zero. It took a long time of patient explanation, of using practical examples and of constant repetition before she came up with her own examples for comparing numbers. Application of mechanical aids showed to be ineffective.

4. Addition and subtraction of whole numbers
It was only after mastering comparisons that we came to practising addition and subtraction of whole numbers. In some cases the model of black and white buttons was applied.

Remedial exercises:
The methodical series included exercises of the following type:

5 + 4	
(-5) + 4	(-5) + 8
5 + (-4)	5 + (-8)
(-5) + (-4)	(-5) + (-8)

5 – 4	4 – 5
(-5) – 4	(-5) - 8
5 - (-4)	5 – (- 8)
(-5) – (-4)	(-5) – (-8)

Michael:

Problem area: Michael has problems with the addition of fractions – he adds all numbers in nominators and all numbers in denominators, e. g.:

$$\frac{1}{2}+\frac{1}{3}+\frac{1}{6}=\frac{3}{11}$$

Generally: $\frac{a}{b}+\frac{c}{d}=\frac{a+c}{b+d}$

Analysis of the problem:
1. comprehension of the concept of fraction as a part of its whole,
2. comprehension of fraction as rational number,
3. comprehension of the addition of fractions,
4. equivalence of fractions,
5. looking for a common denominator,
6. determining the least common multiple of given numbers.

Work with models:
Four equal circles from coloured paper, work with drinks (lemonade), measures for liquid. The child has to participate actively in the activity.

A) We shall divide the coloured circles: the first one into two equal parts, the second one into three equal parts, the third one into six equal parts, the forth circle into eleven equal parts. We shall cut out the parts.
We shall put together one half, one third and one sixth – these parts form the whole circle. We shall compare it with three elevenths. Obviously, they are not equal.

B) We take a measure of the volume of 1 liter, and we pour there one half of litre of lemonade, then one third of litre and one sixth of litre – we compare it with three elevenths of a litre of lemonade. This concrete activity with pouring lemonade had a great effect.

7. Remedial exercises:

a) Addition of fractions with the same denominators, e. g. $\frac{1}{5}+\frac{3}{5}$
b) Addition of fractions with such denominators that one is a multiple of the other, e. g. $\frac{1}{4}+\frac{3}{8}$
c) Addition of fractions with relatively prime denominators, e. g. $\frac{3}{5}+\frac{2}{7}$
d) Addition of fractions with denominators having the greatest common denominator greater than 1, e. g. $\frac{3}{8}+\frac{5}{12}$

To denote the exercises formally a table expressing single steps of the addition proved to be very useful:

exercise	common denominator	equivalent fractions	result
$\frac{4}{5}+\frac{7}{9}$	$\frac{\quad}{45}$	$\frac{36}{45}+\frac{35}{45}$	$\frac{71}{45}=1\frac{26}{45}$
$\frac{7}{12}+\frac{5}{9}$	$\frac{\quad}{36}$	$\frac{21}{36}+\frac{20}{36}$	$\frac{41}{36}=1\frac{5}{36}$

10. Practising numeration

Numbers and numeric operations

1. Choose three different single-digits different from zero. (Example: 3, 5, 8.) Write down all three-digit numbers consisting of the chosen single-digits so that every single-digit is used only once in every number. (In our example 358, 385, 538, 583, 835, 853.)
Calculate the sum of the obtained three-digit numbers. (In our example: 358 + 385 + 538 + 583 + 835 + 853 = 3 552.) Divide this sum by the sum of the original single-digits. (In our example 3 552 ÷ 16 = 222.) The result is always 222.
This exercise is suitable for a class of learners with mixed maths abilities and skills, e. g. dyscalculic individuals have problems with noting down the six three-digit numbers, bright learners can explain why the result is always 222.)

2. Choose one of the numbers in Exercise 1 – such that the number of units and the number of hundreds differ at least by two. (E. g. 835.) Write down the number with the reverse sequence of digits (538). Subtract the smaller number from the bigger one. (835 – 538 = 297). Reverse the digits of this number (792) and calculate the sum of these two numbers. (297 + 792 = 1 089) The result is always 1 089.

3. A) Choose a three-digit number and write it down twice to get the six-digit number. (E. g. 538 538.) Divide this number by 13, the result by 11 and then by 7. If you do not make a mistake you obtain the original three-digit number.
(538 538 ÷ 13 = 41 426, 41 426 ÷ 11 = 3 766, 3 766 ÷ 7 = 538)
B) Multiply numbers 13, 11 and 7. What is interesting on the obtained number? Multiply it by the three-digit number from Exercise A. Compare the results of Exercises A and B.

4. Write down the number giving your age (a two-digit number) three times to get the six-digit number. Divide this number by 13, the result by 21 and then by 37. If you do not make a mistake, you will obtain the original number.

5. Notice interesting products obtained by multiplying:
   a) multiply 37 by multiples of three: 37 x 3, 37 x 6, etc.
   b) multiply 3 367 by multiples of three.
   c) multiply 15 873 by multiples of seven.

6. Multiply 12 345 679 by any single-digit different from one. Multiply the result by nine. What do you notice?

7. a) The sum of natural numbers 1 through 10 is 55. Estimate and then calculate the product of numbers 1 x 2 x ... x 10.
   b) Determine the sum of all natural numbers 1 through 100.
   c) Determine the sum of all odd natural numbers 1 through 100.
   d) Determine the sum of all even natural numbers 1 through 100.

8. Self-confidence test: evaluate by numbers 1 to 10 (10 being the best) your ability to do your present job, add twice the number of points evaluating your ability to do your boss's job and then add three times the number of points you think your boss would evaluate your ability to

do your job. This sum is your evaluation number we shall work with: add one, multiply the result by two, take away three, divide by four, add five, multiply by six, take away 7.5, divide by three, and take away your evaluation number. If your result is seven, you are excelent, if it is not, you are probably too self-confident, you could do better at least in maths.

Basic concepts of statistics

1. We work with a class as with a statistical population, the learners are statistical units. Every learner is given a small cube. We prepare a "mat" with statistical inquiry, e. g.

Group A: shoe size
Group B: results from a test.

Task: A – place your cube onto the position of your shoe size.
B – place your cube onto the position what was your result from the test.

By means of cubes we create a culumn chart and explain the basic concepts:

statistical population, its size, statistical sample
statistical unit
quantitative and qualitative data
absolute frequency
relative frequency
frequency distribution and its graphical representation (histogram, frequency polygon, pie chart, etc.)
mode
median
arithmetic mean
weighted mean, geometric mean, harmonic mean, their practical meaning
variance, standard deviation

2. Notice the properties of arithmetic mean:

a) Five people have the following salaries: 6 000 CZK, 7 000 CZK, 8 000 CZK, 9 000 CZK, 10 000 CZK. The arithmetic mean is 8 000 CZK.
b) Another group of five people has the salaries: 6 000 CZK, 7 000 CZK, 80 000 CZK, 9 000 CZK, 10 000 CZK. The arithmetic mean is 26 500 CZK.
c) The salaries of other five people are: 5 000 CZK, 20 000 CZK, 30 000 CZK, 40 000 CZK, 50 000 CZK. The arithmetic mean is 29 000 CZK.

The arithmetic mean is influenced by extreme values.

Building-up probability reasoning

We practice the following concepts:

random experiment, the set of all possible results of an experiment
event, certain event, impossible event
the probability of event A

1. We throw a six-sided die once, we note down the number of points of single outcomes. We repeat the experiment fifty times.

1	2	3	4	5	6

We illustrate the difference between statistical frequency and theoretical probability.

2. We throw two six-sided dice which are possible to distinquish. We note down the sums of both dice. We repeat the experiment fifty times.

Sum	Possibilities					
2	1 1					
3	1 2	2 1				
4	1 3	2 2	3 1			
5	1 4	2 3	3 2	4 1		
6	1 5	2 4	3 3	4 2	5 1	
7	1 6	2 5	3 4	4 3	5 2	6 1
8	2 6	3 5	4 4	5 3	6 2	
9	3 6	4 5	5 4	6 3		
10	4 6	5 5	6 4			
11	5 6	6 5				
12	6 6					

How many possibilities are there?
What is the probability of throwing certain sums?
What is the absolute and relative frequency of the experiments?

3. We throw six dice. Which of the following events is more probable?
   a) At least two dice will have the same number of points.
   b) Each die will have a different number of points.
   c) There will be neither a), nor b).

4. We throw three dice. Determine the sum which will come up most often.

5. There are three balls in a box – two red ones and one white. We draw one ball at random, note down its colour and put it back into the box. We repeat this experiment fifty times. We calculate absolute and relative frequency of the event "we pick a red ball" and compare it with the theoretical probability.

6. We flip two coins. What is the probabilty that both come up heads?

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