COMPUTATIONAL ALGORITHMS FOR NATURAL NUMBERS

English summary

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Part A.

The text presents detailed methodical instructions for trainers how to approach the subject matter of written addition, subtraction, multiplication and division. Deducing each of these operations is structured into several elementary steps in such a way that every following example includes only one new element. This kind of teaching enables trainees to understand every operation thoroughly because it consistently and systematically presents examples in the order of their difficulty, from easier ones to more complicated ones.

The introductory part explains the notion of algorithm as a precise and unambiguous set of instructions for solving a particular problem in a step-by-step procedure which is possible to describe by a finite text. Algorithm is able to transform the elements of a certain input set (input objects) into output objects. The notion of algorithm is not only a mathematical concept, people meet algorithm in many situations, e.g. as instructions for using various devices, procedure of performing various activities, etc. Algorithms are used on a large scale in programming.

In school mathematics, algorithms are indispensable in teaching written operations with natural and decimal numbers. The algorithms of computing operations in the field of natural numbers give an exact step-by-step procedure so that it could be used generally and consisted of a finite set of elementary steps leading towards the correct result. If the operations with natural numbers are mastered well, then mastering the operations with decimal numbers becomes easier.

The text emphasizes the difference between written and mental methods because mental methods always use calculating from left to right, while written methods (except for written division) calculate from right to left. In order to master written algorithms a person must thoroughly know and understand basic connections of single mental operations (addition, subtraction, multiplication tables, division, division with remainder). A trainee then applies the ability to choose a certain strategy, to respect a way of denoting the operations and to do the single elementary steps in the correct order.

The second part is devoted to deducing written addition, subtraction, multiplication and division. Written addition is deduced in two steps:

a) addition without carrying tens
b) addition with carrying tens.

Deducing the algorithm is shown in the case of two-digit numbers, then it is generalized for numbers of more digits. The procedure is divided into elementary steps and the result of each example is checked by addition of the numbers in changed order.

Written subtraction is also first presented on examples without carrying tens and then a considerable attention is paid to the explanation of written subtraction with carrying tens (based on the principle that adding equal quantities to the minuend and the subtrahend leaves the difference unaffected – we add ten units to the minuend and one ten to the subtrahend). The result is checked by addition.

Written multiplication starts by multiplication by a single digit. The algorithm is practised on the examples where we do not carry amounts to other place value, and then the examples with carrying amounts are presented. Multiplication by a two-digit number is deduced in two steps – multiplication by a multiple of ten (20, 30, etc.) and then by any two-digit numbers (23, 45, etc.)

The algorithm for written division is specific because it is performed from left to right and because its denoting (the form of algorithm) is complicated. Moreover, it is necessary to know all mental operations. The explanation is structured into a fine methodical line when each example teaches only one new activity:

1. the divisor is included in the first digit of the dividend, division is without a remainder (68 : 2)
2. the divisor is included in the first digit of the dividend, but the number of tens is a number which is not divisible by the divisor without a remainder (54 : 2)
3. the divisor is not included in the first digit of the dividend (156 : 3)
4. Division with a remainder (1756 : 3)
5. Division of numbers in which some digits are zeroes.

Written division by a two-digit number is deduced analogously. The attention is paid to estimating the quotient and checking the result.

The next part presents suggestions and topics of exercises for practising, differentiated work, and interesting problem solving exercises.

Part B.

Problems with learning algorithms for written operations.

The text points out possible difficulties which one can meet when mastering algorithms of written operations. They include:

a) numbers are written into the columns incorrectly
b) misunderstanding of the positional decimal system
c) misunderstanding of the calculation procedure in single steps
d) problems with mental calculations
e) problems with changing the procedure (e.g. a part of the exercise is done as addition, but another part as subtraction)
f) choosing a wrong algorithm (e.g. using the procedure of addition for multiplication)
g) problems with numbers having zeroes in some digits.

Part C.

Exercises for differentiated practising of mathematical subject matter

The text includes exercises for practising numeration and numeric operations in the field of natural numbers up to one hundred and practical exercises. Exercises of various types are presented in three modified versions giving instructions concerning the same problem. The first type consists of basic subject matter, the following exercises are more difficult, they can be used also for further education.

Part D.

Exercises with environmental problematics

It is suitable to practise operations with natural and decimal numbers on applied exercises whose content extends students' knowledge. The exercises make use of data concerning inter-disciplinary relationships, e.g. astronomy, biology, chemistry, physics, geography. Numerical information from these areas thus enables practising mathematical subject matter on interesting examples.