PERIMETERS AND AREAS

OF GEOMETRIC FIGURES

English Summary

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The text is based on basic notions relating to the general concept of measure of a geometric figure and on the instructions for specific manipulative activities which help a better comprehension of the relationships.

1. Basic notions

The basic notions are presented in the following way, according to the Dictionary of School Mathematics (Slovník školské matematiky):

The measure of a shape or a figure is a common name for the length of a shape on a line, or a curve, for the area of a shape in the plane, or on a surface area, for the volume of a solid in the space. The notion of measure expresses common properties of functions which assign shapes non-negative real numbers for their lengths (areas, volumes).

Let a system M of shapes be given in such a space P in which the congruence of shapes and the notions of internal and boundary elements of a shapes are defined. A measure of shapes of the system M is a function *m* having the following properties:

1. For every shape X a real number $m(X)$ is assigned such that $m(X) \geq 0$.
2. For every two congruent shapes X, Y such numbers $m(X)$, $m(Y)$ are assigned that
$$m(X) = m(Y).$$
3. For every two shapes X, Y which have no common internal element in space P such numbers $m(X)$, $m(Y)$ are assigned that $m(X \cup Y) = m(X) + m(Y)$.
4. There exists at least one shape E such that $m(E) = 1$.

Length of a line is a number assigned to a line segment by a measure *m* defined on the set of all line segments. (Also the size of a line, the distance of two points.)

Perimeter of a shape is the length of a curve or a broken-line which is the border of a shape in the plane. The measurability of a boundary is one of the typical features of shapes.

Area of a shape in the plane is a number assigned to a shape by a measure *m* defined on a certain set of planar shapes. $m(E) = 1$ is assigned for a selected square E.

In school mathematics we first deduce areas of orthogons whose sides are whole numbers, which are possible to cover with equal squares E, then areas of parallelograms, triangles, trapeziums, polygons. Areas of other shapes are determined by means of their kernels and hulls.

Volume of a solid is a number assigned to a solid by a measure *m* defined on a certain set of solids. $m(B) = 1$ is assigned to a selected cube B.

2. Deducing perimeters and areas of geometric shapes

All relationships are deduced on the basis of concrete activities. The trainer is provided with instructions for constructivist teaching - the work is done by trainees who are thus active when obtaining knowledge, and the lesson is free of passive conveying of knowledge without any trainees' efforts.

A) Rectangle and Square

Perimeter of a rectangle and square
We start by a manipulative activity and practical examples – determining the perimeter of concrete rectangles and squares. We come to the generalizations:

Rectangle:
We add the lengths of all sides: $o = a + b + a + b$
We make use of the equal length of the opposite sides: $o = 2a + 2b$
We make use of the addition of adjacent sides: $o = 2(a + b)$

Square:
We add the lengths of all sides: $o = a + a + a + a$
We make use of the equal length of all sides: $o = 4a$

Area of a rectangle and square
Manipulative activities are based on covering a rectangle whose sides are expressed by natural numbers with squares the area of which is 1 cm^2. After understanding the multiplication of the number of lines by the number of columns of a concrete rectangle, we get a generalization: The area of a rectangle with sides a, b:

$$S = a \, . \, b$$

Squares whose sides are expressed by natural numbers are covered analogously and the area of a square with the side-length a is generalized:

$$S = a \, . \, a = a^2$$

Parallelogram
The perimeter is deduced analogously to the perimeter of a rectangle. The area of a parallelogram is deduced by transforming a parallelogram with sides a, b to a rectangle whose one side is a and the other side is the height of the parallelogram corresponding to side a (or side b, and the corresponding height).
The area of a rhombus is illustrated by another possibility – extending the rhombus so as to form a rectangle of the double area by means of the diagonals.

Triangle
The perimeter of a triangle is based on the sum of the lengths of its sides:

$$o = a + b + c.$$

The area of a triangle is deduced from the area of a parallelogram - the parallelogram of sides a, and its corresponding height v_a is divided into two equal triangles. The area of a triangle is then a half of the product of the side and the corresponding height.

Trapezium
The perimeter of a trapezium is given by the sum of the lengths of its sides:

$$o = a + b + c + d.$$

The area of a trapezium can be deduced by several different ways:

a) dividing the trapezium into geometric shapes whose the area we can calculate (triangles, rectangles, parallelograms)
b) transforming into geometric shapes whose area we can calculate
c) extending it to form other geometric shapes.

Regular polygons
The perimeter is equal to the sum of the lengths of all sides of a polygon.
The area of a regular polygon is usually calculated as the sum of the areas of isosceles triangles with the base being the length of the polygon and the corresponding height being the radius of the circumscribed circle for the polygon.

Circle
The circumference (perimeter) of a circle is determined in an experiment – we measure the circumferences o and the corresponding diameter d of various circles and we calculate the ratio o : d. We calculate the ration approximately 3,14 and we deduce the formula for the circumference of a circle: $o = 2 \pi r$.
The area of a circle is approximated by transforming the circle to a parallelogram – the circle is cut into sectors (16 or 32 the best) which are then put into the shape of the parallelogram. The following formula is then deduced:

$$S = \pi r^2$$

The formulas for sections and segments of a circle are deduced analogously.

3. Reasoning – half of work

There are suggestions for exercises in which it is easy to determine the areas of geometric shapes if we think the task through well and divide the shape into suitable parts. Thus we do not have to calculate much.

4. Exercises

Applied exercises for practicing calculating areas and perimeters of geometric shapes. There are interesting mathematical tasks in which we can creatively apply the relations deduced before and also applied exercises from real life or sports.

5. Suggestions of projects

There are several suggestions for independent work. Trainees are to prepare projects which can be applied into practice, they implement interdisciplinary relationships and actively practice the mathematical subject matter.

6. Several interesting exercises

Exercises using the generalization of Pythagoras' theorem, calculating the areas of interesting shapes composed of basic geometric shapes.